Problem 1, Solution

Since each handshake involves two individuals regardless of order, the number of handshakes is equal to all possible pairs that we can form from a group of 20, which is equal to 20 choose 2. The total number of handshakes is therefore (20) (19) /2=190.

Problem 2, Solution

There are 54 different cards, so the number of 5- card poker hand is C_5^{54}. And there are $C_3^4 C_2^4$ hands which contain 3 Aces and 2 Queens (why?).

The probability of a hand containing 3 Aces and 2 Queens is $\frac{C_3^4 \times C_2^4}{C_5^{54}}$ =0.00000759

Problem 3, Solution

Since there are 3 wheels and 12 different characters in every wheel, in total there will be 12^3 different combinations. If we want to win, we have to get 2 "X" and 1 "XX", so there are C_2^3 possible cases. Then the probability that we win is:

$$P(A)=\frac{C_2^3}{12^3}=\frac{3}{12^3}=0.00174$$

3.9 (a) Let m be number of tails $0 \le m \le n$
then number of heads is $n-m$ and the difference is

$$Y = n - m - m = n - 2m \qquad 0 \le m \le n$$

$$\therefore \; S_Y = \{-n, -n+2, \ldots, n-2, n\}$$

(b) $P[Y=0] = P[n=2m] = P[m=\frac{n}{2}]$ for n even.

$P[Y=k] = P[n-2m=k] = P[m=\frac{n-k}{2}]$ for $n-k$ even

3.10 Let $S = \{b_1, b_2, \ldots, b_{2^m}\}$ be the sequence of m-bit passwords as covered by the hacker.
The target system picks a password at random from S.
$X(\zeta)$ is the index of the selected password.

$S_X = \{1, 2, \ldots, 2^m\}$ where the value of X is selected at random from S_X.

$P[i] = 2^{-m} \quad i \in S_X$.

3.49

~~3.32~~ a) Let I_k denote the outcome of the kth Benoulli trials. The probability that the single event occurred in the kth trial is:

$$
\begin{aligned}
P[I_k = 1 | X = 1] &= \frac{P[I_k = 1 \text{ and } I_j = 0 \text{ for all } j \neq k]}{P[X = 1]} \\
&= \frac{P[0\ 0 \ldots \overset{k\text{th outcome}}{1}\ 0 \ldots 0]}{P[X = 1]} \\
&= \frac{p(1-p)^{n-1}}{\binom{n}{1} p(1-p)^{n-1}} = \frac{1}{n}
\end{aligned}
$$

Thus the single event is equally likely to have occurred in any of the n trials.

b) The probability that the two successes occurred in trials j and k is:

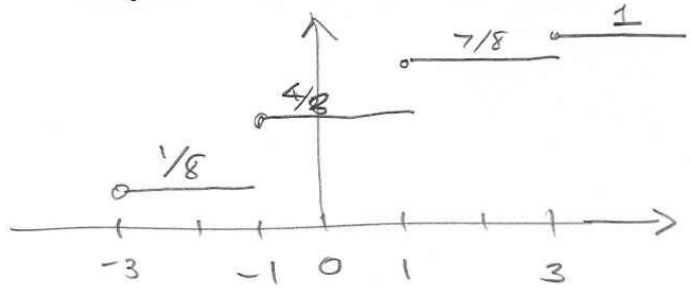
$$
P[I_j = 1, I_k = 1 | X = 2] = \frac{P[I_j = 1, I_k = 1, I_m = 0 \text{ for all } m \neq j, k]}{P[X = 2]}
$$

4.5 $Y = N_H - N_T = N_H - (n - N_H) = 2N_H - n$

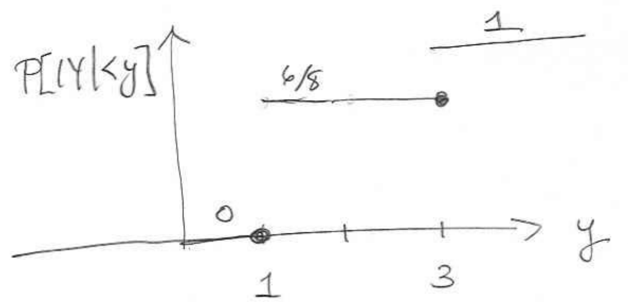
$n = 3$

$$P[N_H = k] = \binom{3}{k}\left(\frac{1}{2}\right)^3$$

n_H	0	1	2	3
$y(n_H)$	-3	-1	1	3
$P[y]$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

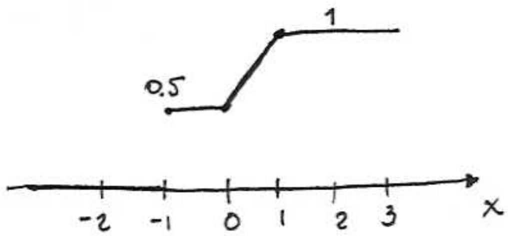


$$P[|Y| < y] = P[-y < Y < y] = F_Y(y^-) - F_Y(-y)$$



4.12

a)



Mixed type random variable

b) $P[X \leq -1] = 0.5$

$P[X = -1] = 0.5$

$$P[X < 0.5] = P[X \leq 0.5] - P[X = 0.5]$$
$$= \frac{1 + 0.5}{2} - 0$$
$$= 0.75$$

$$P[-0.5 < X < 0.5] = P[X \leq 0.5] - P[X = 0.5] - P[X \leq -0.5]$$
$$= \frac{1 + 0.5}{2} - 0 - 0.5$$
$$= 0.75 - 0.5$$
$$= 0.25$$

$$P[X > -1] = 1 - P[X \leq -1]$$
$$= 1 - 0.5$$
$$= 0.5$$

$P[X \leq 2] = 1$

$$P[X > 3] = 1 - P[X \leq 3]$$
$$= 1 - 1$$
$$= 0$$

4.14

a) Mixed Type

b) $P[X<-1]=0$

$P[X\leq -1]=\frac{2}{10}$

$$P[-1<X<-0.75]=P[X\leq -0.75]-P[X\leq -1]$$
$$=\frac{2}{10}-\frac{2}{10}$$
$$=0$$

$$P[-0.5\leq X\leq 0.5]=P[X\leq 0.5]-P[X\leq -0.5]$$
$$=\frac{8}{10}-\frac{2}{10}$$
$$=\frac{6}{10}$$

$$P[|X-0.5|<0.5]=P[\{X<1\}\cup\{X>0\}]=P[\{0<X<1\}]$$
$$=1-\frac{6}{10}$$
$$=\frac{4}{10}$$