EM Implosion Memos

Memo 9

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Lens Design for a Prolate-Spheroidal IRA

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Abstract

The basic design consideration for the physical concept of three different types of increasing permittivity dielectric lens are considered .

1 Introduction

Three types of increasing permittivity dielectric lens design are considered for a prolate-spheroidal IRA that is based on [1].

1.1 Description of geometry

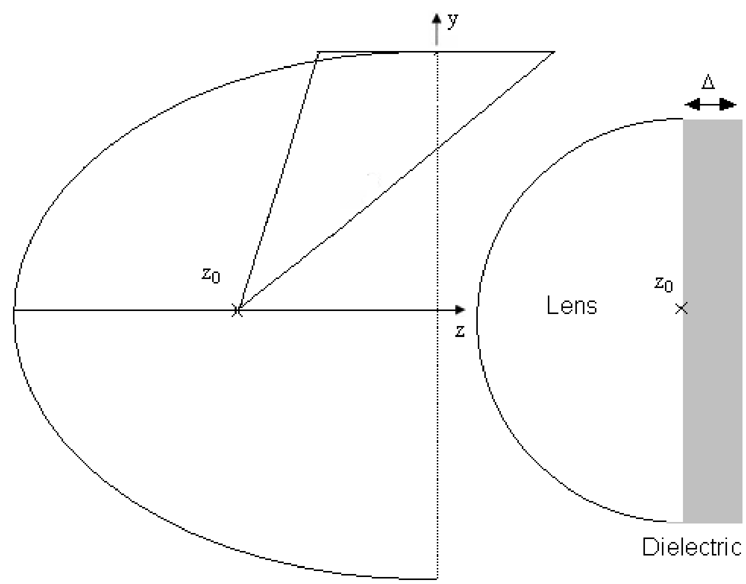

Figure 1.1 IRA Geometry [1]

The focal point is $z_0 = 37.5$ cm and the other parameters of prolate-spheroidal IRA is defined in [2,3].The lens is a half sphere(or half ball in mathematicians terms) and its radius is r_{max} .

2. Design Considerations

As discussed in [4] before, the exponential variation of the characteristic impedance of a transmission line along the line is optimal, provided the speed of propagation is constant along the line. Some modification may be useful here since the speed varies inversely with the square root of the dielectric constant.

The lens relative permittivity is

$$\varepsilon_r(r) = \begin{cases} 1 & \text{at } r = r_{max} \\ \varepsilon_{r\,max} & \text{at } r = 0 \end{cases} \tag{2.1}$$

3. Exponential Variation of ε_r

One suitable form for ε_r is an exponential function as

$$\varepsilon_r(r) = e^{q(r_{max}-r)} \tag{3.1}$$

As we know at $r=0$ the relative permittivity is $\varepsilon_{r=}\varepsilon_{r\,max}$ so

$$\varepsilon_{r\,max} = e^{C_1(r_{max})}$$
$$C_1 = \frac{1}{r_{max}}\ln(\varepsilon_{r\,max}) \tag{3.2}$$

if we substitute (2.3) in (2.1), ε_r can be found as

$$\varepsilon_r(r) = e^{\ln(\varepsilon_{r\,max})(1-\frac{r}{r_{max}})} \tag{3.3}$$

The rise time is estimated as $t_\delta = 100\,\mathrm{ps}$ so the distance corresponding to this rise time is

$$\ell_\delta = c\,t_\delta = 3\,\mathrm{cm} \tag{3.4}$$

in air.

The propagation time of the wave from $r = r_{max}$ to $r = 0$

$$ct_{lens} = c\int_0^{r_{max}} \frac{1}{v}\,dr = \int_0^{r_{max}} \varepsilon_r^{1/2}(r)\,dr = \int_0^{r_{max}} e^{\frac{1}{2}\ln(\varepsilon_{r\,max})(1-\frac{r}{r_{max}})}\,dr$$
$$= (\varepsilon_{r\,max}^{1/2}-1)\frac{2\,r_{max}}{\ln(\varepsilon_{r\,max})} \tag{3.5}$$

Normalized ct_{lens} is

$$\frac{ct_{lens}}{r_{max}} = (\varepsilon_{r\,max}^{1/2}-1)\frac{2}{\ln(\varepsilon_{r\,max})} \tag{3.6}$$

The distance between source and lens is $(.375\,\mathrm{m} + r_{max})$ .The impulsive term has to go an extra $2x\,.25\,\mathrm{m}$ distance between the first focal point to the reflector.

After all of this design procedure we designed a lens that is matched to the target dielectric $\varepsilon_{r\,max}$.

The thickness of the target dielectric material should be

$$\Delta = n\frac{c\,t_\delta}{\sqrt{\varepsilon_{r\,max}}}\ ,\ n \ge 2 \tag{3.7}$$

to minimize the effect of the reflected wave on the impulsive term.

4.Compensated Incremental Speed (CIS) form of ε_r

As we mentioned before the exponential form assumes that propagation speed is constant but it is not constant so we are trying to compensate this assumption. Let us assume we have a plane wave problem in an inhomogeneous (isotropic) slab with $\varepsilon_r(z)$ and set relative change in wave impedance over a transit time $\Delta\tau$

$$\frac{\Delta \ln(\varepsilon_r^{-1/2})}{\Delta\tau} = C_2 \tag{4.1}$$

wave impedance Z_c is proportional to $\varepsilon_r^{-1/2}$. Distance based on the transit time can be written as

$$c\,d\tau = \varepsilon_r^{1/2}\,dz \tag{4.2}$$

For a given $\Delta\tau$ the Δz decreases as $\varepsilon_r^{-1/2}$. For a given $\Delta\tau$ the change in $\ln(\varepsilon_r^{-1/2})$ is independent of z and if we substitute (4.2) in (4.1)

$$\varepsilon_r^{-1/2}\frac{d\ln(\varepsilon_r^{-1/2})}{dz} = C_2 \tag{4.3}$$

So let us integrate (5.3)

$$\begin{gathered}\int \varepsilon_r^{-1/2} d\ln(\varepsilon_r^{-1/2}) = \int e^{\ln(\varepsilon_r^{-1/2})}\,d\ln(\varepsilon_r^{-1/2}) = \int C_2\,dz \\ \varepsilon_r^{-1/2} = C_2 z + C_3\end{gathered} \tag{4.4}$$

We can define ε_r from (5.4) as

$$\varepsilon_r = \left(C_2 z + C_3\right)^{-2} = \begin{cases} 1 & \text{at } z = z_{max} \\ \varepsilon_{r\,max} & \text{at } z = 0 \end{cases} \tag{4.5}$$

so one can find from (5.5)

$$C_2 = \frac{1-\varepsilon_{r\,max}^{-1/2}}{z_{max}},\ C_3 = \varepsilon_{r\,max}^{-1/2} \tag{4.6}$$

if we substitute (5.6) in (5.5)

$$\varepsilon_r = \left((1-\varepsilon_{r\,max}^{-1/2})\frac{z}{z_{max}} + \varepsilon_{r\,max}^{-1/2}\right)^{-2} \tag{4.7}$$

How much time does the propagation of the wave take from r_{max} to focal point in the lens? So let's substitute (5.7) in (3.6)

$$ct_{lens} = \int_0^{r_{max}} \varepsilon_r^{1/2}\,dr = \int_0^{r_{max}} \left((1-\varepsilon_{r\,max}^{1/2})\frac{r}{r_{max}} + \varepsilon_{r\,max}^{-1/2}\right)^{-1} dr \tag{4.8}$$

let us change the variable of the integral as $\xi = r/r_{max}$ so we will have

$$ct_{lens} = r_{max}\int_0^1 \left((1-\varepsilon_{r\,max}^{-1/2})\xi + \varepsilon_{r\,max}^{-1/2}\right)^{-1} d\xi \tag{4.9}$$

let us change the variable of the integral as $\varsigma = (1-\varepsilon_{r\,max}^{-1/2})\xi + \varepsilon_{r\,max}^{-1/2}$ so

$d\varsigma = (1-\varepsilon_{r\max}^{-1/2})\,d\xi$ and we will have

$$ct_{lens} = \frac{r_{max}}{1-\varepsilon_{r\max}^{-1/2}} \int_{\varepsilon_{r\max}^{-1/2}}^{1} (\zeta)^{-1}\, d\zeta = \frac{1}{2}\frac{r_{max}}{1-\varepsilon_{r\max}^{-1/2}} \ln(\varepsilon_{r\max}) \qquad (4.10)$$

so we can normalized ct_{lens} is

$$\frac{ct_{lens}}{r_{max}} = \frac{1}{2}\frac{1}{1-\varepsilon_{r\max}^{-1/2}} \ln(\varepsilon_{r\max}) \qquad (4.11)$$

5. Linear form of ε_r

Exponential Variation and CIS form of ε_r are two different approaches have some advantages and disadvantages about focusing. After these approaches we are trying to use another approach which is linearly increasing form of ε_r. Let us assume we have a linear ε_r variation as

$$\varepsilon_r(r) = r/r_{max} + \varepsilon_{r\max}\left(1 - r/r_{max}\right) \qquad (5.1)$$

which satisfies (2.1), so we can find the normalized propagation time of the wave from $r = r_{max}$ to $r = 0$ as

$$\frac{ct_{lens}}{r_{max}} = \frac{1}{r_{max}} \int_0^{r_{max}} \varepsilon_r^{1/2}(r)\, dr = \int_0^{r_{max}} \left(r/r_{max} + \varepsilon_{r\max}\left(1 - r/r_{max}\right)\right)^{1/2} dr \qquad (5.2)$$

Let us change the variable of this integral as $\zeta = r/r_{max}$ so we will have

$$\frac{ct_{lens}}{r_{max}} = \int_0^1 \left[\zeta + \varepsilon_{r\max}(1-\zeta)\right]^{1/2} d\zeta \qquad (5.3)$$

We can also change this variable ζ as

$$\begin{aligned} \xi &= \zeta + \varepsilon_{r\max}[1-\zeta] \\ d\xi &= d\zeta\left[1 - \varepsilon_{r\max}\right] \end{aligned} \qquad (5.4)$$

so we will have from (5.3)

$$\frac{ct_{lens}}{r_{max}} = \frac{1}{1-\varepsilon_{r\max}} \int_{\varepsilon_{r\max}}^{1} \xi^{1/2} d\xi = \frac{2}{3}(\varepsilon_{r\max} - 1)^{-1}(\varepsilon_{r\max}^{3/2} - 1) \qquad (5.5)$$

6.Conclusion

Dielectric exponentially increasing, CIS and linear increasing lens designs are discussed.

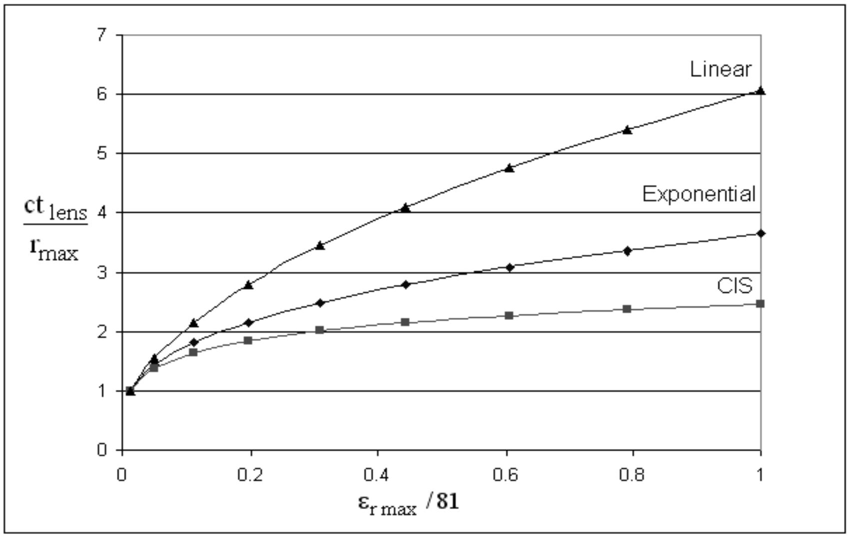

Figure 5.1 ct_{lens} / r_{max} for Linear, Exponential and CIS forms of ε_r

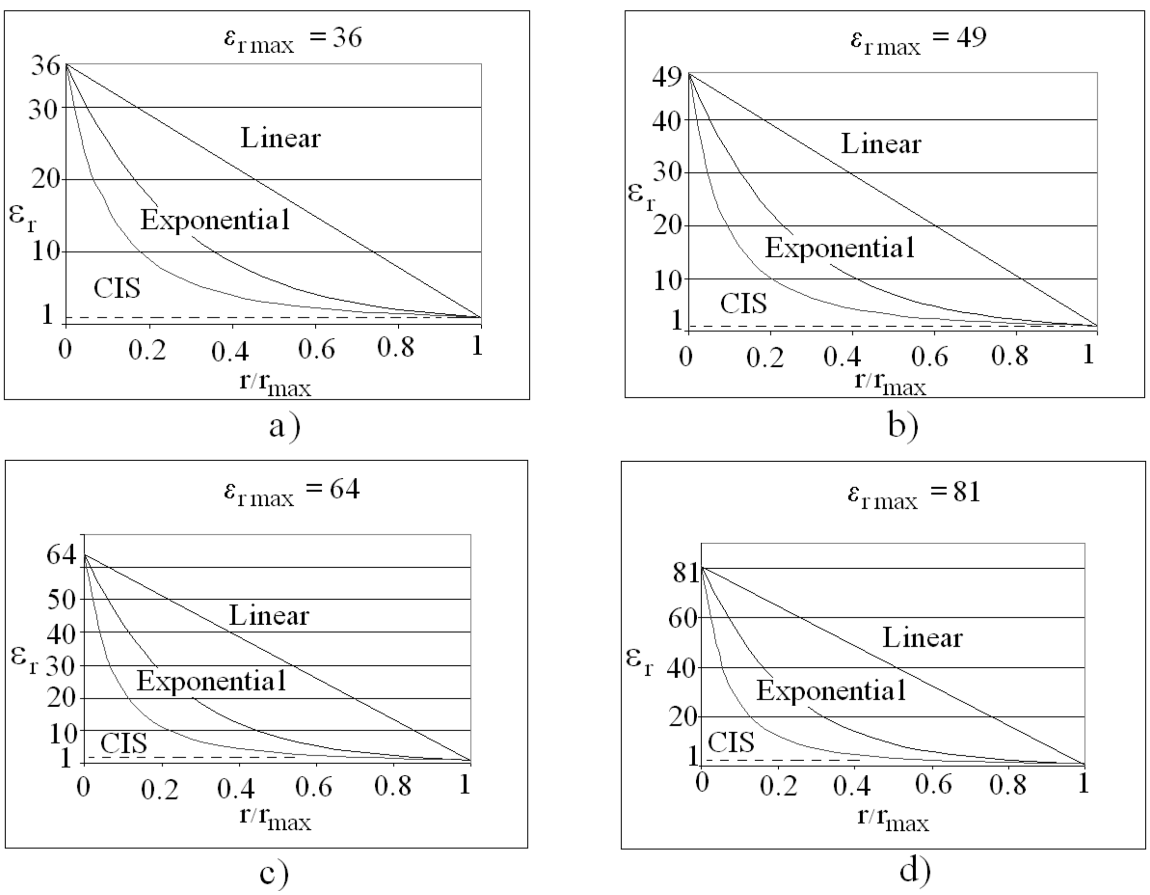

Figure 5.2 a) ε_r for Linear, Exponential and CIS forms of ε_r

One can see from fig. 5.1 that the wave propagates faster for the CIS form of ε_r. We can see from fig. 5.2 a-d) that If $\varepsilon_{r\,max}$ increases the wave propagates slower as expected. $\varepsilon_{r\,max}$ varies from 1 to 81 (corresponding to water which is the highest ε_r that is used in biological applications). If we increase $\varepsilon_{r\,max}$ from 36 to 81, the CIS design of ε_r has the deepest curvature. The focusing property of the lens increases from the CIS to the linear design, because for the same r/r_{max} we have an increase in ε_r so we expect the lens to become more effective. Also from [1] if we increase ε_r the spot size decreases while the wave impedance decreases and the amplitude of the waveform increases with the ratio of $\varepsilon_r^{1/4}$. This rough calculation has to extend out some distance from the target for an effective focusing to occur and this requires more detailed computations.

References

1. C. E. Baum, "Addition of a Lens Before the Second focus of a Prolate-Spheroidal IRA " Sensor and Simulation Note 512, April 2006.
2. C. E. Baum, "Focal Waveform of a Prolate-Spheroidal IRA", Sensor and Simulation Note 509, February 2006.
3. S. Altunc and C. E. Baum, "Extension of the Analytic Results for the Focal Waveform of a Two-Arm Prolate-Spheroidal Impulse-Radiating Antenna (IRA)", Sensor and Simulation Note 518, Nov 2006.
4. C. E. Baum and J. M. Lehr, "Nonuniform-Transmission-Line Transformers for Fast High-Voltage Transients",Circuit and Electromagnetic System Design Note 44, February 2000; IEEE Trans. Plasma Science, 2002, pp. 1712-1721.