Measurement Notes

Note 39

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An Anisotropic Medium for High Wave Impedance

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Abstract

This paper considers the use of special geometrical hybrids of dielectric and magnetic materials in which the resultant permittivity is dominated by the dielectric medium, and the resultant permeability is dominated by the magnetic medium. This gives some control over the wave impedance and propagation constant for electromagnetic waves, both for transient lenses and for high wave impedances for chokes (for baluns and transformers). In general these media are anisotropic such that the magnetic field in the magnetic material is not interrupted by breaks in the layers or rods of such material. The electric field, being orthogonal to the magnetic field for the waves of interest, is purposely made to be orthogonal to such layers or rods so that the dielectric medium dominates for this field.

I. Introduction

Artificial dielectrics have long been used to construct lenses for microwave antennas [5]. Typically one embeds metallic pieces (disks, spheres, wires, etc.) in a dielectric medium. This increases the effective permittivity for wavelengths large compared to the spacing of these pieces. There is also some decrease in the effective permeability due to the exclusion of the magnetic field by the conductors. By this technique one can significantly decrease the propagation speed (at frequencies of interest in the lens) below that of free space

$$c \equiv [\mu_o \epsilon_o]^{-\frac{1}{2}} \tag{1-1}$$

One often uses a regular spacing (cubical, tetrahedral) or random spacing of such conductors with sufficient symmetry so as to achieve isotropic permittivities and permeabilities.

In lens design one may also be concerned with increasing the permeability so that the wave impedance is more like that of free space

$$Z_o = \left[\frac{\mu_o}{\epsilon_o}\right]^{\frac{1}{2}} \tag{1.2}$$

to avoid reflections at the lens surface in a broad-band or transient sense [8], one increases the permeability by the use of ferrites or other magnetic materials. These are used in transformer cores and various microwave devices [6]. Besides the permeability such materials may also have undesirable properties which lower the wave impedance, such as conductivity and permittivity (greater than ϵ_0).

II. Basic Uniform Isotropic Medium

Let a uniform isotropic medium be characterized by

$$\begin{aligned} \mu &\equiv \mu_r \mu_o \equiv \text{permeability} \\ \epsilon &\equiv \epsilon_r \epsilon_o \equiv \text{permittivity} \\ \sigma &\equiv \text{conductivity} \end{aligned} \tag{2.1}$$

where these may be a function of the complex frequency (two-sided Laplace transform variable (over time))

$$s \equiv \Omega + j\omega \tag{2.2}$$

The wave impedance is now

$$\tilde{Z} = \left[\frac{s\mu}{\sigma + s\epsilon}\right]^{\frac{1}{2}} = Z_o \left[\frac{\mu_r}{\epsilon_r + \frac{\sigma}{s\epsilon_o}}\right]^{\frac{1}{2}} \tag{2.3}$$

and the propagation constant is

$$\tilde{\gamma} = [s\mu(\sigma + s\epsilon)]^{\frac{1}{2}} = \frac{s}{c}\left[\mu_r\left(\epsilon_r + \frac{\sigma}{s\epsilon_o}\right)\right]^{\frac{1}{2}} \tag{2.4}$$

where a tilde over a quantity indicates a function of s (Laplace transformed).

Looking at the wave impedance note that for zero conductivity we have

$$\tilde{Z} = Z_o \sqrt{\frac{\mu_r}{\epsilon_r}} \tag{2.5}$$

for which case we readily have the free space wave impedance as Z_o if $\mu_r = \epsilon_r$, or more generally from (2.3) if

$$\mu_r = \epsilon_r + \frac{\sigma}{s\epsilon_o} \tag{2.6}$$

(noting that μ_r can be frequency dependent and complex). In this case (2.4) becomes

$$\tilde{\gamma} = \frac{s}{c}\,\mu_r = \frac{s}{c}\left[\epsilon_r + \frac{\sigma}{s\epsilon_o}\right] \tag{2.7}$$

If μ_r and ϵ_r are constant with $\sigma = o$ then such a medium just has a slower propagation speed. The problem in this case is to find or synthesize such a medium, say for lens use. Note that the presence of non zero σ, if too large, can distort the passage of a pulse. This also points out that we would like ϵ_r and μ_r to be approximately frequency independent for frequencies of interest.

Another case of interest has

$$|\mu_r| >> \left|\epsilon_r + \frac{\sigma}{s\epsilon_o}\right| \tag{2.8}$$

$$|\tilde{Z}| >> Z_o$$

which has application for inductors (chokes) and transformers. In this case one may not be too concerned if $\tilde{Z}$ is a little frequency dependent as long as it is large for frequencies of interest. The requirement then is for large μ_r without a corresponding large $\epsilon_r + \sigma/s\,\epsilon_o$.

III. Dielectric and Magnetic Sandwich

Now consider a hybrid anisotropic medium as indicated in Figure 3.1. This consists of an alternating set of layers

$$\text{type 1 (dielectric):} \quad \mu_o \ , \ \epsilon_1$$

$$\text{type 2 (magnetic):} \quad \mu_2 \ , \ \epsilon_2 \ , \ \sigma_2$$

$$\epsilon_1 \equiv \epsilon_{r_1} \epsilon_o \ , \ \mu_2 \equiv \mu_r \epsilon_o \ , \ \epsilon_2 \equiv \epsilon_{r_2} \epsilon_o$$

$$d = d_1 + d_2 \tag{3.1}$$

$$\Delta_1 \equiv \frac{d_1}{d} \ , \ \Delta_2 \equiv \frac{d_2}{d}$$

$$\Delta_1 + \Delta_2 = 1$$

For present purposes we restrict the magnetic field to be parallel to the y axis with the electric field parallel to the x axis, appropriate to waves propagating in the $\pm z$ direction. The layers are parallel to the $y\,z$ plane (perpendicular to the x axis). Note that the magnetic field in the magnetic material is not interrupted by the dielectric medium, but the electric field in the magnetic medium is interrupted by the dielectric spacers. Then under certain circumstances the dielectric material can dominate the dielectric (and conducting) properties while the magnetic material can dominate the magnetic properties. While one can perform a full wave analysis of propagation in such a medium, let us for present purposes assume that d is electrically small, i.e., that radian wavelengths and skin depths as appropriate in both layers are small compared to their respective thicknesses.

For the electric field E_x a pair of adjacent layers acts as series capacitors/resistors. Noting that the total current density (conduction plus displacement) is continuous in the x direction through the boundaries, we have

$$J_{t_1} = J_{t_2} = s\epsilon_1 E_1 = (\sigma_2 + s\epsilon_2) E_2 \tag{3.2}$$

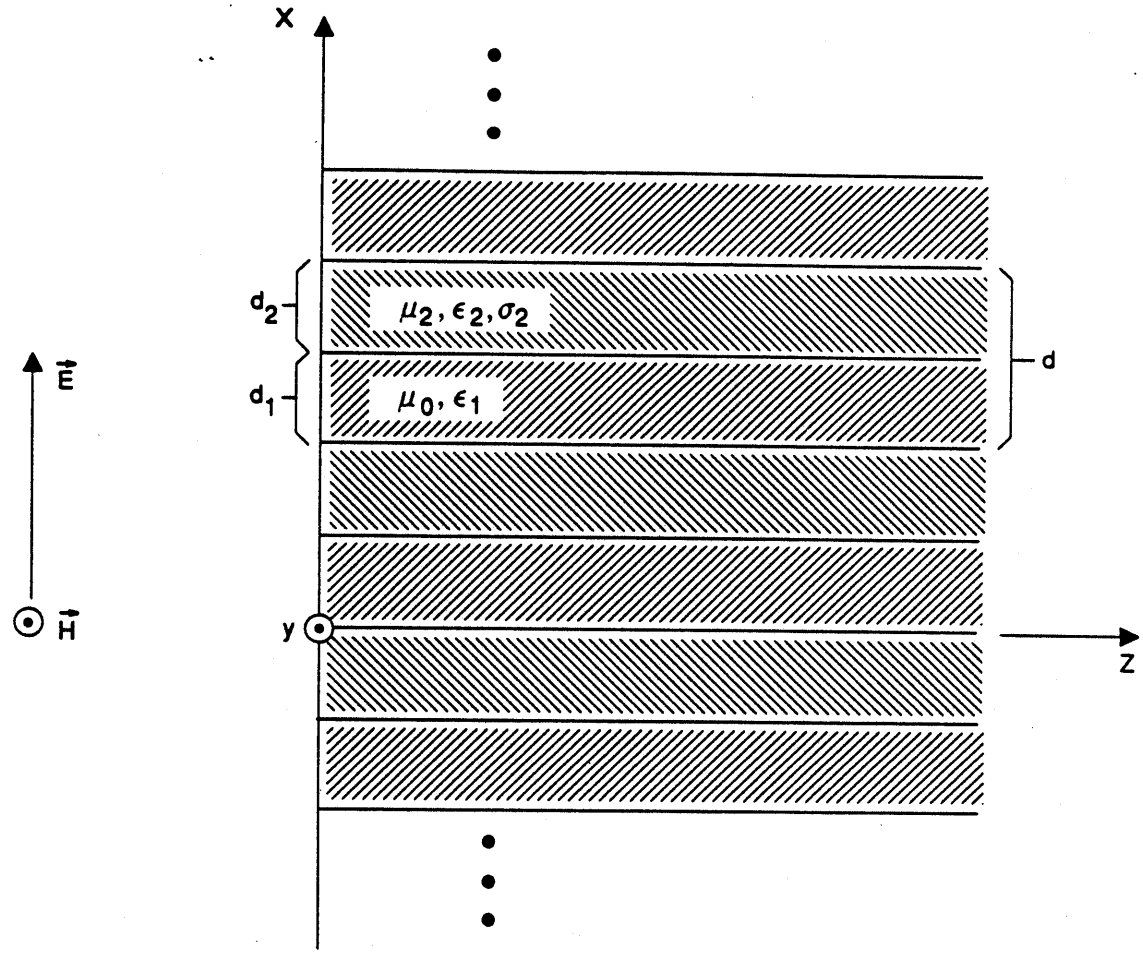

Figure 3.1. Dielectric/Magnetic Sandwich in a Half Space

where subscripts indicate the respective layers. The average electric field is

$$E_{avg} = \frac{1}{d}\left[d_1 E_1 + d_2 E_2\right] = \Delta_1 E_1 + \Delta_2 E_2 \tag{3.3}$$

from which we have the effective conduction and dielectric properties as

$$\begin{aligned} \frac{1}{\sigma + s\epsilon} &= \frac{E_{avg}}{J_t} = \frac{\Delta_1 E_1 + \Delta_2 E_2}{s\epsilon_1 E_1} = \frac{\Delta_1}{s\epsilon_1} + \frac{\Delta_2}{s\epsilon_1}\frac{E_2}{E_1} \\ &= \frac{\Delta_1}{s\epsilon_1} + \frac{\Delta_2}{s\epsilon_2 + \sigma_2} \end{aligned} \tag{3.4}$$

Consider limiting cases. First for negligible σ_2 we have

$$\frac{1}{\epsilon} = \frac{\Delta_1}{\epsilon_1} + \frac{\Delta_2}{\epsilon_2} \tag{3.5}$$

as the usual capacitive divider. Second for large σ_2 so that the electric field is shorted out in medium 2 we have

$$\frac{1}{\epsilon} = \frac{\Delta_1}{\epsilon_1} \tag{3.6}$$

showing that in this case medium 1 dominates the dielectric properties.

For the magnetic field H_y a pair of adjacent layers acts as parallel inductors. Noting that the magnetic field (H_y) is continuous in the x direction through the boundaries we have

$$H_1 = H_2 = \frac{B_1}{\mu_o} = \frac{B_2}{\mu_2} \tag{3.7}$$

The average magnetic flux density (B) is

$$B_{avg} = \frac{1}{d}\left[d_1 B_1 + d_2 B_2\right] = \Delta_1 B_1 + \Delta_2 B_2 \tag{3.8}$$

from which we have the effective magnetic property as

$$\begin{aligned}\mu &= \frac{B_{avg}}{H} = \frac{\Delta_1 B_1 + \Delta_2 B_2}{H} = \Delta_1 \mu_o = \Delta_2 \mu_2 \\ &= \mu_o\left[\Delta_1 + \Delta_2 \mu_r\right]\end{aligned} \tag{3.9}$$

For large μ_r then medium 2 dominates the magnetic properties.

Referring to Figure 3.1 consider a wave impingent on the $z = 0$ plane from the left. The effective wave impedance looking into the special medium is

$$\tilde{Z} = \left[\frac{s\mu}{\sigma + s\epsilon}\right]^{\frac{1}{2}} = Z_o\left\{\left[\Delta_1 + \Delta_2 \mu_r\right]\left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2} + \frac{\sigma_2}{s\epsilon_o}}\right]\right\}^{\frac{1}{2}} \tag{3.10}$$

The propagation constant is

$$\tilde{\gamma} = [s\mu(\sigma + s\epsilon)]^{\frac{1}{2}} = \frac{s}{c}\left\{[\Delta_1 + \Delta_2\,\mu_r]\left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2} + \frac{\sigma_2}{s\epsilon_o}}\right]^{-1}\right\}^{\frac{1}{2}} \tag{3.11}$$

Assuming negligible σ_2 the wave impedance is

$$\tilde{Z} = \left[\frac{\mu}{\epsilon}\right]^{\frac{1}{2}} = Z_o\left\{[\Delta_1 + \Delta_2\,\mu_r]\left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}}\right]\right\}^{\frac{1}{2}} \tag{3.12}$$

With $\Delta_1 = 1 - \Delta_2$ let us maximize the quantity in braces by setting the derivative with respect to Δ_1 equal to zero as

$$\begin{aligned} 0 &= [1-\mu_r]\left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{1-\Delta_1}{\epsilon_{r_2}}\right] + [\Delta_1 + (1-\Delta_1)\,\mu_r]\left[\frac{1}{\epsilon_{r_1}} - \frac{1}{\epsilon_{r_2}}\right] \\ &= \frac{2\Delta_1}{\epsilon_{r_1}} + \frac{1-2\Delta_1}{\epsilon_{r_2}} + \mu_r\left\{\frac{1-2\Delta_1}{\epsilon_{r_1}} - \frac{2(1-\Delta_1)}{\epsilon_{r_2}}\right\} \end{aligned} \tag{3.13}$$

Then the wave impedance is maximized for large μ_r with

$$\begin{aligned} &\frac{1-2\Delta_1}{\epsilon_{r_1}} = \frac{2-2\Delta_1}{\epsilon_{r_2}} \\ &\Delta_1 = \frac{1}{2}\,\frac{\epsilon_{r_2} - 2\,\epsilon_{r_1}}{\epsilon_{r_2} - \epsilon_{r_1}} \quad , \quad \Delta_2 = \frac{1}{2}\,\frac{\epsilon_{r_2}}{\epsilon_{r_2} - \epsilon_{r_1}} \end{aligned} \tag{3.14}$$

Now let $\epsilon_{r_2} \geq 2\epsilon_{r_1}$ for the solution to apply (an acceptable range of Δ_1 , Δ_2). Then we have

$$\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}} = \frac{1}{2\epsilon_{r_1}} \tag{3.15}$$

or $\mu_r \to \infty$ the wave impedance is then

$$\tilde{Z} = Z_o \left\{ \frac{\Delta_1 + \Delta_2 \mu_r}{2 \epsilon_{r_1}} \right\}^{\frac{1}{2}} = \frac{Z_o}{2} \left\{ \frac{\epsilon_{r_2}(\mu_r + 1) - 2\epsilon_{r_1}}{\epsilon_{r_1}(\epsilon_{r_1} - \epsilon_{r_1})} \right\}^{\frac{1}{2}}$$

$$= \frac{Z_o}{2} \left\{ (\mu_r + 1) \frac{\epsilon_{r_2}}{\epsilon_{r_1}} \frac{1}{\epsilon_{r_2} - \epsilon_{r_1}} \right\}^{\frac{1}{2}} \tag{3.16}$$

Similarly the propagation constant is

$$\tilde{\gamma} = \frac{s}{c} \left\{ (\Delta_1 + \Delta_2 \mu_r) 2 \epsilon_{r_1} \right\}^{\frac{1}{2}} = \frac{s}{c} \left\{ \frac{\epsilon_{r_2}(\mu_r + 1) - 2\epsilon_{r_1}}{\epsilon_{r_2} - \epsilon_{r_1}} \epsilon_{r_1} \right\}^{\frac{1}{2}}$$

$$= \frac{s}{c} \left\{ (\mu_r + 1) \left[\frac{1}{\epsilon_{r_1}} - \frac{1}{\epsilon_{r_2}} \right] \right\}^{\frac{1}{2}} \tag{3.17}$$

For large μ_r then with maximizing choice of Δ_1 we see that medium 2 dominates the inductive part.

Assuming now large ϵ_{r_2} with negligible σ_2 we have from (3.13) for maximum wave impedance

$$\Delta_1 = \frac{1}{2} \frac{\mu_r}{\mu_r - 1} \quad , \quad \Delta_2 = \frac{1}{2} \frac{\mu_r - 2}{\mu_r - 1} \tag{3.18}$$

so we assume $\mu_r \geq 2$ for this solution to apply. The corresponding wave impedance is

$$\tilde{Z} = \frac{Z_o}{2} \left\{ \frac{\mu_r}{\epsilon_{r_1}} \frac{\mu_r}{\mu_r - 1} \right\}^{\frac{1}{2}} \tag{3.19}$$

The propagation constant is

$$\tilde{\gamma} = \frac{s}{c} \left\{ (\mu_r - 1) \epsilon_{r_1} \right\}^{\frac{1}{2}} \tag{3.20}$$

For large ϵ_{r_2} then with maximizing choice of Δ_1 we have medium 1 dominating the capacitive part.

Looking at these results, we can let both μ_r and ϵ_{r_2} be large, showing from (3.12) that we have a good choice for large wave impedance as

$$\Delta_1 = \Delta_2 = \frac{1}{2}$$

$$\tilde{Z} = \frac{Z_o}{2}\left\{\frac{\mu_r + 1}{\epsilon_{r_1}}\left[1 + \frac{\epsilon_{r_1}}{\epsilon_{r_2}}\right]\right\}^{\frac{1}{2}} \tag{3.21}$$

The propagation constant is similarly

$$\tilde{\gamma} = \frac{s}{c}\left\{(\mu_r + 1)\epsilon_{r_1}\left[1 + \frac{\epsilon_{r_1}}{\epsilon_{r_2}}\right]^{-1}\right\}^{\frac{1}{2}} \tag{3.22}$$

So equal thickness layers are rather reasonable giving the dielectric layer dominating the capacitive parts and the magnetic layer dominating the inductive parts.

Including some conductive losses in the magnetic material we need to expand the propagation constant from (3.11) for large s as

$$\left\{\frac{\Delta_1}{\epsilon_{r_2}} + \frac{\Delta_2}{\epsilon_{r_2} + \frac{\sigma_2}{s\epsilon_o}}\right\}^{-\frac{1}{2}} = \left\{\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}}\left[1 - \frac{\sigma_2}{s\epsilon_2} + O(s^{-2})\right]\right\}^{-\frac{1}{2}}$$

$$= \left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}}\right]^{-\frac{1}{2}}\left\{1 + \frac{1}{2}\left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}}\right]^{-1}\frac{\sigma_2}{s\epsilon_2} + O(s^{-2})\right\}$$

(3.23)

$$\tilde{\gamma} = \frac{s}{c}\left\{[\Delta_1 + \Delta_2\,\mu_r]\left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}}\right]^{-1}\right\}^{\frac{1}{2}}$$

$$+ \frac{\sigma_2}{2c\epsilon_2}[\Delta_1 + \Delta_2\,\mu_r]^{\frac{1}{2}}\left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}}\right]^{-\frac{3}{2}} + O(s^{-1})$$

For a wave propagating as $e^{-\tilde{\gamma} z}$ the first term gives the delay part that we have already considered (for $\sigma_2 = 0$). The second term gives the attenuation. So write

$$\tilde{\gamma} z = sT + \alpha z + O(s^1) \tag{3.24}$$

The delay is

$$T \equiv \frac{z}{v} = \frac{1}{c}\left[\Delta_1 + \Delta_2 \mu_r\right]^{\frac{1}{2}} \left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}}\right]^{-\frac{1}{2}}$$

$$v = c\left[\Delta_1 + \Delta_2 \mu_r\right]^{-\frac{1}{2}} \left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}}\right]^{\frac{1}{2}} \equiv \textit{speed} \tag{3.25}$$

The attenuation is represented by a transmission factor

$$f = e^{-\alpha z} \tag{3.26}$$

which is ideally near 1 (or small αz), at least for lens use. From (3.23) we have

$$\alpha z \equiv \frac{z}{z_o} = \frac{\sigma_2}{2c\,\epsilon_2}\left[\Delta_1 + \Delta_2 \mu_r\right]^{\frac{1}{2}} \left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}}\right]^{-\frac{3}{2}}$$

$$= \frac{\sigma_2}{2\epsilon_2}\, T \left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}}\right]^{-1} \tag{3.27}$$

with z_o as an attenuation length. In time units we have

$$\alpha z \equiv \frac{T}{T_o}$$

$$T_o = 2\frac{\epsilon_2}{\sigma_2}\left[\frac{\Delta_1}{\epsilon_{r_1}} + \frac{\Delta_2}{\epsilon_{r_2}}\right] \tag{3.28}$$

where T_o is now related to the relaxation time in medium 2 with some improvement if $\epsilon_{r_1} << \epsilon_{r_2}$. For negligible attenuation the thickness of the layered medium in the z direction should have the transit time small compared to T_o.

IV. Cylindrical Waves with C_∞ Symmetry

This special layered medium is also applicable to special kinds of cylindrical waves. As illustrated in Figure 4.1 let the same layered medium be extended over all space except for the volume within a cylinder of radius a centered on the z axis. The layers are now taken as parallel to the x , y plane (perpendicular to the z axis). The layers have the same parameters as in Section III. Note the use of cylindrical (Ψ, ϕ, z) coordinates with

$$x = \Psi \cos(\phi) \quad , \quad y = \Psi \sin(\phi) \tag{4.1}$$

The configuration in Figure 4.1 is taken as independent of ϕ (C_∞ symmetry).

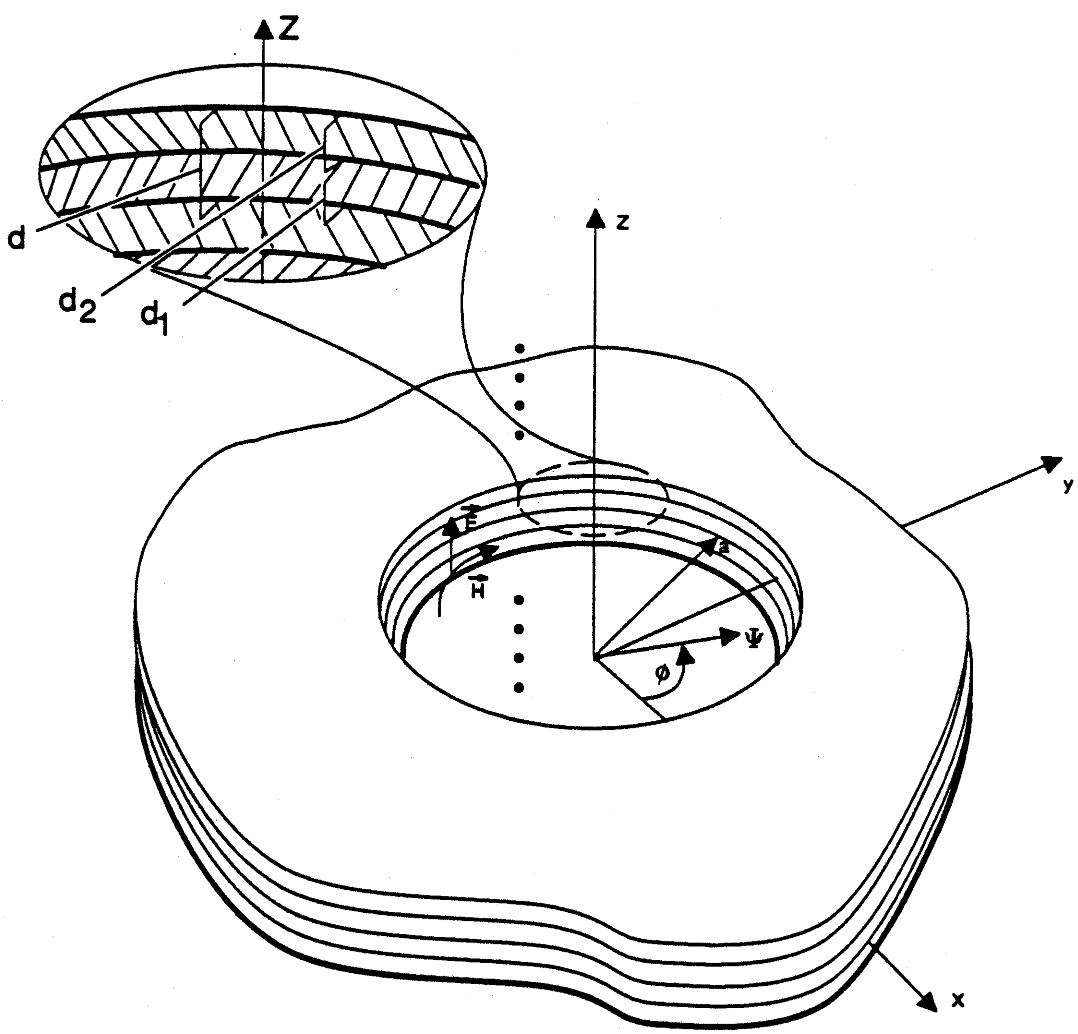

Figure 4.1. Dielectric/Magnetic Sandwich with C_∞ Symmetry

This particular geometry is appropriate for a choke that might be used for a pulse transformer or balun [3,4]. In this case a set of cables with shields bonded together has a gap region where the shields are broken and the inner conductors are cross connected to each other and the shields in various ways. Pulses are then transmitted in an impedance-matched fashion through the gap region, incoming on certain cables and outgoing on others. The purpose of the choke is to prevent currents from flowing on the outside of the cable shields away from the gap region. Consider that the cable bundle with common outer shield is placed along the z axis and can be considered to have some radius $b < a$. Then the exterior shield currents have a magnetic field in the ϕ direction, i.e. parallel to the layers as desired. The associated electric field has Ψ and z components, so think of the choke as a set of alternating dielectric and magnetic washers with the cable-gap region somewhere in the middle.

The geometry in Figure 4.1 is then an idealization of such a choke. As a further simplification let the currents near the z axis be z directed and independent of both z and ϕ. The resulting fields have only E_z and H_ϕ components. These fields meet the same restrictions for the analysis in Section III to apply, except that now we have cylindrical instead of plane waves. Note that for present calculations the choke is taken of infinite extent in the $\pm z$ directions and infinite radially as $a \le \Psi < \infty$. Furthermore, radian wavelengths and skin depths in the various media are assumed large compared to d so that the average parameters (wave impedance and propagation constant) in the layered media are applicable.

With these restrictions on the field components we can use the usual expansions of the fields in cylindrical coordinates for outgoing waves [1,2]. The z and ϕ independent solution has for $\Psi > a$

$$E_z = E_o \, K_o(\tilde{\gamma} \Psi)$$

$$H_\phi = -\frac{E_o}{\tilde{Z}} \, K_1(\tilde{\gamma} \Psi) \tag{4.2}$$

This gives an impedance per unit length (of the core in the z direction) as

$$\tilde{Z}' = -\frac{1}{2\pi a}\frac{E_z}{H_\phi}\bigg|_{\Psi=a} = \frac{\tilde{Z}}{2\pi a}\frac{\mathrm{K}_o(\tilde{\gamma}a)}{\mathrm{K}_1(\tilde{\gamma}a)} \tag{4.3}$$

with $\tilde{\gamma}$ and $\tilde{Z}$ as in the previous section. For large arguments of the modified Bessel functions [7] we have

$$\mathrm{K}_o(\tilde{\gamma}a) = \left[\frac{\pi}{2\tilde{\gamma}a}\right]^{\frac{1}{2}}[1 + O((\tilde{\gamma}a)^{-1})]$$

$$\mathrm{K}_1(\tilde{\gamma}a) = \left[\frac{\pi}{2\tilde{\gamma}a}\right]^{\frac{1}{2}}[1 + O((\tilde{\gamma}a)^{-1})] \tag{4.4}$$

$$\tilde{Z}' = \frac{\tilde{Z}}{2\pi a}[1 + O((\tilde{\gamma}a)^{-1})]$$

which is just the wave impedance divided by circumference as expected. For small arguments we have

$$\mathrm{K}_o(\tilde{\gamma}a) = \left\{-\ln\left(\frac{\tilde{\gamma}a}{2}\right) + C_e\right\}[1 + O((\tilde{\gamma}a)^2)]$$

$$C_e = .5772156649\ldots \equiv \text{Euler's constant}$$

$$\mathrm{K}_1(\tilde{\gamma}a) = \frac{1}{\tilde{\gamma}a}[1 + O((\tilde{\gamma}a)^2)]$$

$$\tilde{Z}' = \frac{\tilde{\gamma}}{2\pi}\left\{-\ln\left(\frac{\tilde{\gamma}a}{2}\right) + C_e\right\}[1 + O((\tilde{\gamma}a)^2)] \tag{4.5}$$

Using the low-frequency form of $\tilde{\gamma}$ as in (3.11) we can see that this is basically inductive as we should expect.

In terms of frequencies on the $j\omega$ axis we have the alternate representations [8]

$$\tilde{\gamma} = j\tilde{k} \quad , \quad s = j\omega$$

$$\mathrm{K}_o(j\tilde{k}a) = -j\frac{\pi}{2} H_o^{(2)}(\tilde{k}a) \tag{4.6}$$

$$\mathrm{K}_1(j\tilde{k}a) = -\frac{\pi}{2} H_1^{(2)}(\tilde{k}a)$$

The impedance per unit length is then

$$\tilde{Z}' = \tilde{Z}\frac{j}{2\pi a}\frac{H_o^{(2)}(\tilde{k}a)}{H_1^{(2)}(\tilde{k}a)} \tag{4.7}$$

which at low frequencies becomes

$$\tilde{Z}' = \frac{j\tilde{k}}{2\pi}\left\{-\ln\left(\frac{\tilde{k}a}{2}\right) - j\frac{\pi}{2} + C_e\right\}\left[1 + O\left((\tilde{k}a)^2\right)\right] \tag{4.8}$$

V. Extension to Waves with One Magnetic Orientation but Two Possible Electric Orientations

With the basic concept elaborated, let us consider a simple variation on the theme. As discussed in Section III, the layers are arranged so that the magnetic field is not interrupted in the magnetic layers, while an insulating dielectric (of low dielectric constant) is used to interrupt the electric current density flowing from one magnetic layer to the next. This basic idea can be extended as indicated in Figure 5.1. Instead of layers, consider rods of magnetic material parallel to the y axis. With only an H_y the magnetic field in each rod is not interrupted by any gaps. Letting there be both x and z components of the electric field, note the dielectric isolation of the rods in both these directions. With identical radii b, and spacing of centers by d in both x and z directions, the wave propagation and impedance properties will be the same for waves propagating in both these directions, and in general for waves propagation in any direction perpendicular to the y axis. Of course we assume that radian wavelengths and skin depths are large compared to both d and b.

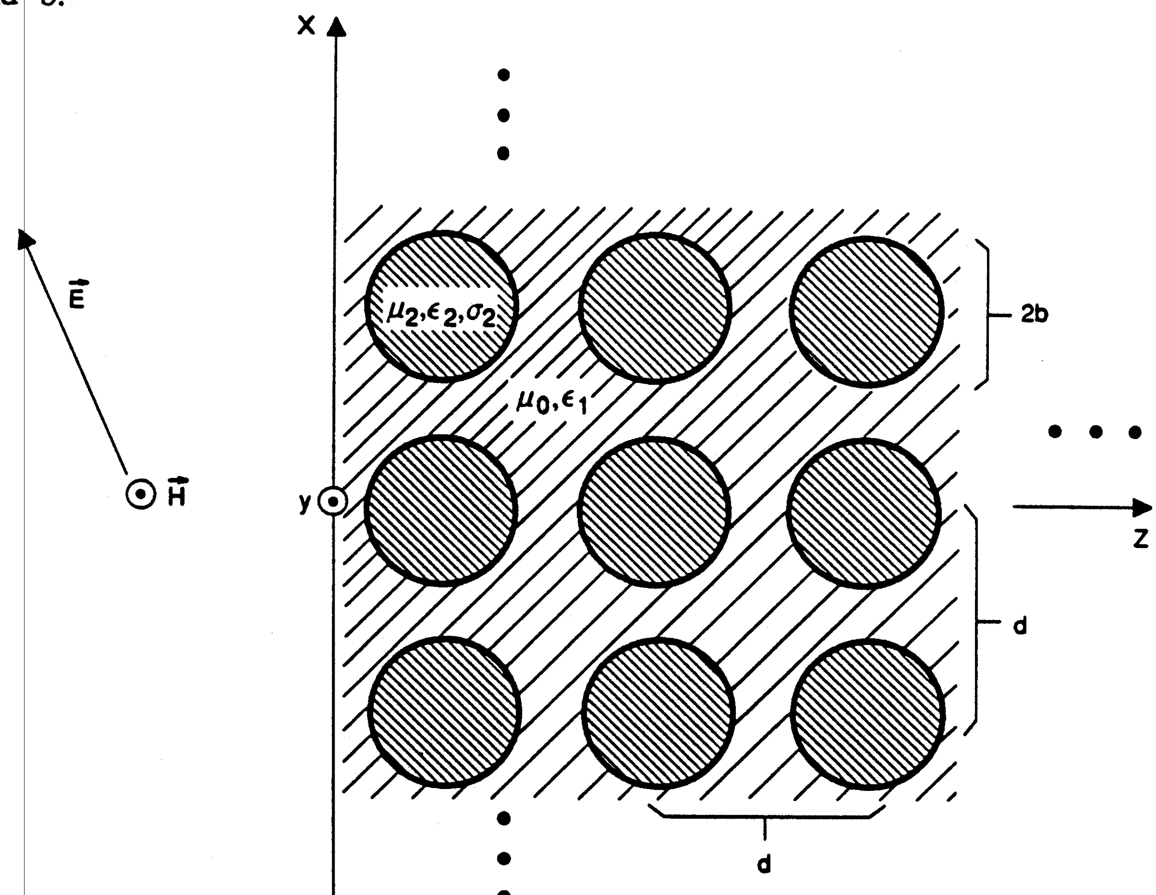

Figure 5.1. Magnetic Rods Embedded in Dielectric Medium

The calculation of the effective wave impedance and propagation constant is somewhat more complicated than in the case of layers as in Section III. Here one can take a unit cell of dimensions $d \times d$ (or $d/2 \times d/2$ using symmetry) and solve for the effective μ and ϵ (with σ) using quasistatic techniques (both magnetic and electric). Of course, the rods need not be circular but should have the same symmetry as the unit cell. Furthermore a square unit cell is not essential. An equilateral triangular unit cell with rods centered on the corners could also be used.

The arrangement in Figure 5.1 is appropriate for plane waves. One can generalize this in a manner analogous to Figure 4.1 by taking magnetic rings all with axis as the z axis. Then on a plane of constant ϕ a cross section would look like Figure 5.1. Such a configuration is appropriate for fields with components H_ϕ, E_z, E_Ψ in cylindrical coordinates, H_ϕ, E_r, E_θ in spherical coordinates with

$$\Psi = r\cos(\theta) \quad , \quad z = r\sin(\theta) \tag{5.1}$$

Then with currents on the cable shield(s) near the z axis we do not need to restrict the fields to be z independent for the model of the choke in Section IV to apply. This more general form of C_∞ symmetry allows spherical as well as cylindrical waves to be considered due to the allowance of two electric components with H_ϕ. Note that for the simple quasistatic model of a unit cell to apply to all such cells, the cell dimension d should be small compared to the local radius Ψ for radii of interest in this hybrid medium (choke).

VI. Concluding Remarks

This paper has explored some techniques for combining dielectric and magnetic media for control of the effective parameters of the hybrid medium. By configuring the different media so that the permittivity (and conductivity) is dominated by the dielectric medium, and the permeability is dominated by the magnetic medium, one can achieve some control over the wave impedance and propagation constant.

The geometries appropriate to chokes (high wave impedance) admit of simple layers of such media with more elaborate rod configurations possible. For transient-lens geometries the desired wave impedance may be more like free space [8]. However, the associated TEM mode pattern can have a not-so-simple magnetic-field orientation which varies in space. The sheets or rods of magnetic material may then ideally need to be tapered in thickness and bent in shape to match this field pattern.

Here we have basically addressed geometric ideas. The requisite dielectric and (especially) magnetic media are another matter. While the geometries introduce additional possibilities, one still needs to use practical materials which may still have some undersirable limitations.

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