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ON THE RESOLUTION OF TWO NEARLY COINCIDENT PULSES PROPAGATED THROUGH A DISPERSIVE CHANNEL

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ABSTRACT

Through the use of a simple model, the propagation of two nearly coincident pulse modulated sinusoidal signals through a dispersive channel is considered. A simple expression is derived for the minimum time separation required for signal resolution in terms of parameters characterizing the source function, the channel bandwidth, and the instrumentation bandwidth.

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It is well known that propagation of signals through a dispersive channel such as the ionosphere will modify the shape of the transmitted signal waveform. As the total electron content of the ionosphere increases, the distortion of the waveform becomes more pronounced. The distortion takes the form of a "smearing out" in time of the pulse shape. Therefore, if two distinct signal pulses originally separated in time are propagated through a dispersive channel they will tend to merge. Eventually it becomes impossible to distinguish between the two pulses. This paper concerns the minimum time interval τ, the interval between peak amplitudes, for successful identification, i.e., resolution, of the two pulses.

In order to simplify the analysis it is assumed that the pulses are sinusoidal with gaussian envelopes. Let the field component of one pulse be represented by

$$e(t) = A\, e^{-\alpha^2 t^2}\, e^{j\omega_0 t}, \tag{1}$$

where A is an amplitude factor, α, the damping coefficient, is the effective source bandwidth, and ω_0 is the carrier frequency. Although some other representation may be preferable, choice of Eq. (1) as a source function leads to a remarkably simple expression for τ. It is further assumed that the receiver has a gaussian transfer function of the form

$$G(\omega) = (\sqrt{\pi}/\beta)\, \exp\left[-(\omega - \omega_0)^2/4\beta^2\right], \tag{2}$$

where β is the receiver bandwidth. (This assumption also tends to simplify the analysis.)

Propagation of a single pulse of the form given by (1) through a dispersive channel has been discussed by Wait[1] in a paper on optimum receiver bandwidths. The present paper differs from Wait's treatment in that resolving power theory[2] is used to derive a minimum (not necessarily optimum) bandwidth, β, for resolution of two nearly coincident pulses separated by τ seconds. In the discussion which follows, for convenience the carrier term will be suppressed.

Because of the self-reciprocal nature of the gaussian function, the output, h(t), of the receiver is given by

$$h(t) = g(t) * e(t) = e^{-\beta^2 t^2} * A\, e^{-\alpha^2 t^2} = \frac{\sqrt{\pi\gamma}}{\alpha\beta} A\, e^{-\gamma^2 t^2}, \tag{3}$$

where, by the theorem for the addition of moments,

$$\left(\frac{1}{\gamma}\right)^2 = \left(\frac{1}{\alpha}\right)^2 + \left(\frac{1}{\beta}\right)^2 . \tag{4}$$

If we assume a linear system, the principle of superposition holds and the output of the receiver for two pulses of equal amplitude is

$$h(t) = \frac{\sqrt{\pi\gamma}}{\alpha\beta} A \left[e^{-\gamma^2\left(t - \frac{\tau}{2}\right)^2} + e^{-\gamma^2\left(t + \frac{\tau}{2}\right)^2} \right]. \tag{5}$$

Now the expansion of

$$f\left(t - \frac{\tau}{2}\right) = e^{-\gamma^2\left(t - \frac{\tau}{2}\right)^2} \tag{6}$$

in a Taylor series about t is given by

$$f\left(t - \frac{\tau}{2}\right) = f(t) - \frac{\tau}{2} f'(t) + \frac{1}{8}\tau^2 f''(t) + \dots , \tag{7}$$

and similarly,

$$f\left(t + \frac{\tau}{2}\right) = f(t) + \frac{\tau}{2} f'(t) + \frac{1}{8} \tau^2 f''(t) + \dots . \qquad (8)$$

For very small τ, the series can be truncated beyond the second order term to obtain

$$h(t) = \frac{\sqrt{\pi\gamma}}{\alpha\beta} A\left[2f(t) + \frac{1}{4} \tau^2 f''(t)\right] , \qquad (9)$$

where, of course,

$$f(t) = e^{-\gamma^2 t^2} \qquad (10)$$

and

$$f''(t) = 2\gamma^2 (2\gamma^2 t^2 - 1)\, e^{-\gamma^2 t^2} . \qquad (11)$$

With reference to Eq. (9), if $\frac{1}{4} \tau^2 f''(t)$ is negligible everywhere compared with 2f(0), the two pulses are indistinguishable and have merged to form a single pulse of double intensity at t = 0.

In order for separation of the source pulses to be observed, h(t) must have a central minimum which can occur only if $f''\left(\frac{\tau}{2}\right) > 0$, that is, from Eq. (11), if

$$2\gamma^2 \left(\frac{\tau}{2}\right)^2 > 1 ,$$

or

$$\gamma^2 > 2/\tau^2 . \qquad (12)$$

From (4) the minimum value of β is obtained as

$$\frac{\alpha^2 \beta^2}{\alpha^2 + \beta^2} > \frac{2}{\tau^2} ; \qquad (13)$$

i.e.,

$$\beta > \left(\frac{2\alpha^2}{\tau^2 \alpha^2 - 2} \right)^{1/2} . \qquad (14)$$

Note that, in order to obtain a real β, the condition $\tau > \sqrt{2}/\alpha$ must be satisfied for a receiver with a gaussian passband characteristic (however wide) to resolve two gaussian pulses separated by τ.

In order to extend the discussion to cover the resolution of two distinct pulses propagated through the ionosphere, it is necessary to take into account ionospheric dispersion. If we assume a quadratic phase fit for the approximate transfer function,

$$\Phi(\omega) \simeq \left| \Phi(\omega_0) \right| \exp\left[-j\phi(\omega) \right] , \qquad (15)$$

of the dispersive propagation path in the vicinity of ω_0 it can be shown that the envelope of the field component after transionospheric propagation is proportional to

$$f(t - \tau_g) \propto \exp\left[-\alpha^2 \left(t - \tau_g \right)^2 \left(1 - \frac{\alpha^2}{\alpha^2 - jW^2} \right) \right] , \qquad (16)$$

where τ_g is the group delay and W is the dispersive channel bandwidth given by

$$W = \frac{1}{2} \sqrt{\frac{\omega_0}{\tau_g}} . \qquad (17)$$

The relationship between W and $\phi(\omega)$ has been discussed by Wait[1] and Inston[3]. A numerical value for the group delay is given by the expression

$$\tau_g = \frac{5 \times 10^{-6}}{\omega_0^2} \int_0^s N \, ds . \qquad \text{(MKS)} \qquad (18)$$

The integral represents the integrated electron content (TEC) along the propagation path.

From (16) it is seen that the amplitude of the envelope is proportional to

$$\left|f_R(t - \tau_g)\right| \propto \exp\left[-\alpha^2(t - \tau_g)^2 \left(1 - \frac{\alpha^4}{W^4 + \alpha^4}\right)\right] . \tag{19}$$

For W >> α, the dispersion is negligible and, in comparing Eq. (19) with Eq. (1), it appears that the source signal, after undergoing propagation through the ionosphere, is essentially unmodified in form but delayed by the group delay τ_g. However, for W ~ α, it is apparent that some "smearing out" of the pulse occurs. In this event it is appropriate to select a new damping coefficient α', where

$$\alpha' = \frac{\alpha W^2}{\sqrt{W^4 + \alpha^4}} . \tag{20}$$

By replacing α in (14) by α', the following inequality for β is obtained:

$$\beta > \left[\frac{2\alpha^2 W^4}{\tau^2\alpha^2 W^4 - 2(\alpha^4 + W^4)}\right]^{1/2} . \tag{21}$$

Solving (21) for τ, one finds that

$$\tau^2 > 2\left(\frac{\alpha^2}{W^4} + \frac{1}{\alpha^2} + \frac{1}{\beta^2}\right) . \tag{22}$$

As W increases without limit (22) reduces to (13), as it should.

In most cases of interest it is necessary to determine the minimum τ for fixed α and W, parameters over which the observer has no control. Furthermore, it appears that W must be at least equal to (or greater than) α for the signal to be propagated through the dispersive channel. (Note that the range of values for W is defined by (17) in terms of known τ_g and ω_0.) If we take $W = \alpha$, then (22) reduces to

$$\tau^2 > 2\left(\frac{2}{\alpha^2} + \frac{1}{\beta^2}\right) . \tag{23}$$

Obviously, in this analysis where noise is not a consideration, it is important to maximize the receiver bandwidth, β, the only option open to the circuit designer.

At this point it is informative to consider an illustrative example. Suppose we select a nominal 30-MHz carrier frequency as a reasonable choice for the lowest frequency signal which will be propagated essentially unattenuated through the highest TEC likely to be encountered. Assume, now, a nominal pulse duration of 1 μsec. According to the reciprocal spreading principle ($\Delta f \cdot \Delta t \sim 1$), it is reasonable to take the corresponding damping constant α as 1 MHz. Finally, for ease of computation, let the receiver bandwidth, β, also be 1 MHz. By substituting these values of α and β in (23), the minimum resolution time τ_{min} is found to be 2.45 μsec. Then, with substitution of this value of τ into (19), the corresponding TEC is $3.36 \times 10^{17} m^{-2}$, a value not grossly atypical of an average ionosphere.*

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*The value of $10^{17} m^{-2}$ is commonly taken as typical of an average ionosphere and could vary by at least an order of magnitude in either direction, depending on time of day, season, position in the sunspot cycle, and geographical location.[4]

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