Mathematics Notes

Note 19

May 1970

ERF and ERFC: Mathematical Routines for Computing the Error Function and Complementary Error Function

J. E. Vogel, 9422
Sandia Laboratories, Albuquerque

Abstract

ERF and ERFC are high-accuracy, high-speed routines for computing the error function and complementary error function, respectively. They may also be used to compute various other related functions such as the normal probability integrals.

Key words: Special function

MaN 19

SC-M-70-275

ERF AND ERFC: MATHEMATICAL ROUTINES FOR COMPUTING THE ERROR FUNCTION AND COMPLEMENTARY ERROR FUNCTION

J. E. Vogel, 9422
Sandia Laboratories, Albuquerque

May 1970

ABSTRACT

ERF and ERFC are high-accuracy, high-speed routines for computing the error function and complementary error function, respectively. They may also be used to compute various other related functions such as the normal probability integrals.

Key words: Special function

s

FOREWORD

The Sandia Laboratories Mathematical Program Library consists of a number of dependable, high-quality, general-purpose, mathematical computing routines. The standards established for the library require that these routines be mathematically sound, effectively implemented, extensively tested, and thoroughly documented. This report documents one such routine.

The library emphasizes the effective coverage of various distinct mathematical areas with a minimum number of routines. Nevertheless, it may contain other routines similar in nature but complementary to the one described here. Additional information on the mathematical program library, a description of the standard format for documenting these routines, and a guide to other routines in the library are contained in SC-M-69-337.

This report is also identified within Sandia Laboratories as Computing Publication ML0022/ALL. This report and its corresponding library routines are expected to be available from COSMIC shortly after publication.

CONTENTS

CONTENTS (cont)

ERF AND ERFC: MATHEMATICAL ROUTINES
FOR COMPUTING THE ERROR FUNCTION
AND COMPLEMENTARY ERROR FUNCTION

1. Introduction

1.1 Background

ERF and ERFC grew out of a need for a high-accuracy, high-speed routine to compute the error function and complementary error function. These are the second set of routines provided and are an improvement in speed and accuracy over the previous routines.

1.2 Applicable Programming Languages and Computer Systems

ERF and ERFC are written in CDC 6600 FORTRAN.

The applicable computing system is the Control Data 6600 SCOPE. The routines are maintained for the convenience of the user in a library file. The routines are accessible by means of a few machine-dependent control cards which are described in Appendix C.

1.3 Considerations Regarding Use

ERF and ERFC are to be used for computing values of the error function and complementary error function for any real number. ERF and ERFC may be used to compute various other related functions such as the normal probability integrals. For details on such relations see Appendix D.

2. Usage

2.1 Entry

ERF and ERFC are written as function subprograms and may be referenced anywhere an arithmetic expression is allowed, as on the right-hand side of a FORTRAN replacement statement such as Y = ERF(X). The data types of ERF and ERFC are real.

2.2 Description of Arguments

X Any real variable name, constant, or arithmetic expression. It is an input quantity only and is not altered during execution.

2.3 Restrictions Between Arguments

There are no restrictions between arguments.

2.4 Principal Uses with Examples

The principal use of ERF is to compute $\frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. This can be done in several ways, e.g.,

```
Z = 2.35
Y = ERF(Z),
  or
Y = ERF(2.35).
```

The principle use of ERFC is to compute $\frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$. This can be done in several ways, e.g.,

```
Z = 2.35
Y = ERFC(Z),
  or
Y = ERFC(2.35).
```

2.5 Library Routines Explicitly Required

The standard FORTRAN routines EXP, SIGN, and ABS are used.

2.6 User-Supplied Routines Required

None of the routines called by ERF or ERFC need be supplied by the user.

2.7 Cautions and Restrictions

There are no restrictions on the use of ERF or ERFC. The user is cautioned against using ERF to compute the complementary error function by using the identity ERFC(X) = 1.0 - ERF(X). This subtraction may cause partial or total loss of significance for certain values of X.

2.8 Error Conditions, Messages, and Codes

There are no error conditions, messages, or codes associated with ERF or ERFC.

3. Mathematical Methods

3.1 Statement of Problem

We wish to compute the error function, which is $\frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$, and the complementary error function, which is

$$\frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, dt.$$

3.2 Methods Used

The error function and complementary error function are approximated by rational functions. Three such rational approximations are used depending on whether $|x| < 0.46875$, $0.46875 \le |x| < 4.0$, or $4.0 \le |x|$.

In the first region the error function is computed directly and the complementary error function is computed via the identity $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$. In the other two regions the complementary error function is computed directly and the error function is computed from the identity $\mathrm{erf}(x) = 1.0 - \mathrm{erfc}(x)$.

3.3 Mathematical Range and Domain

The error function and complementary error function are real-valued functions of any real argument. The range of the error function is [-1,1]. The range of the complementary error function is [0,2].

3.4 Equations and Discussion

In the July 1969 issue of Mathematics of Computation, Cody[1] presents certain rational approximations for the error function and complementary error functions. It is well known that the error function and complementary error function are related through the identity

$$\mathrm{erf}(x) + \mathrm{erfc}(x) = 1.0 \ . \tag{1}$$

However, if the error function is computed directly for all values of x and then erfc(x) is computed from relation (1), then a partial or total loss of significance will occur for ranges of x in which $\mathrm{erf}(x) \ge 0.9$, in particular for $|x| > 0.469$. This problem is circumvented in the Cody approximations by presenting rational approximations for erf(x) for $|x| < 0.46875$ and computing erfc(x) from relation (1). For all other values of x, rational approximations are given to compute erfc(x) directly and relation (1) is used to determine the value of erf(x).

The basic technique is to divide the argument range into 3 basic regions. For $|x| \le 0.46875$, erf(x) is given by

$$\mathrm{erf}(x) \approx x \cdot \sum_{i=1}^{4} a_i x^{i-1} \Big/ \sum_{i=1}^{4} b_i x^{i-1} \tag{2}$$

where the coefficients a_i, b_i are given in Reference 1. The complementary error function is then given by erfc(x) = 1.0 - erf(x) .

For $0.46875 \le x \le 4.0$, the complementary error function is computed directly and it is given by

$$\mathrm{erfc}(x) \approx e^{-x^2} \sum_{i=1}^{6} c_i x^{i-1} \Big/ \sum_{i=1}^{6} d_i x^{i-1} \tag{3}$$

where the c_i, d_i are again given in Reference 1. The error function is again given by relation (1). This approximation is good only for positive arguments and so for negative arguments the relations erf(-x) = -erf(x) and erfc(-x) = 2 -erfc(x) are used.

For $4.0 \le x$, a higher order approximation is used to compute erfc(x) and is given by

$$\mathrm{erfc}(x) \approx \frac{e^{-x^2}}{x} \left\{ \frac{1}{\sqrt{\pi}} + \frac{1}{x^2} \left(\sum_{i=1}^{4} f_i \left(\frac{1}{x^2}\right)^{i-1} \Big/ \sum_{i=1}^{4} g_i \left(\frac{1}{x^2}\right)^{i-1} \right) \right\}. \tag{4}$$

Again erf(x) is computed from relation (1), and erf(-x) and erfc(-x) are computed from previously noted identities.

There are in addition to the above approximations slight differences between the approximations used in ERF and ERFC. For certain ranges of x the error function can effectively be approximated by

$$\mathrm{erf}(x) = 1.0 \ \mathrm{sign}\ (x) \ .$$

Likewise, for certain values of x the complementary error function is assigned the value 0.

3.5 Error Analysis, Bounds, Estimates

The error bounds and estimates for ERF and ERFC are primarily empirical; that is, maximum bounds for the error are obtained by testing and comparison with other known values (see Section 6 for further explanation).

The results of these tests indicate that ERF and ERFC, with certain minor exceptions, provide at least 11 significant figures throughout the entire range. The one exception occurs in ERFC for large values of x where the approximation ERFC(x) = 0 is used. No significant figures are obtained in this region; however, the absolute error is less than 1×10^{-294}.

4. Programming Method

The implementation of the equations of Section 3.4 is done in an obvious and straightforward manner. For the region $|x| < 0.46875$ the rational approximation is a ratio of 3rd degree polynomials. For the region $0.46875 < x \le 4.0$ the rational approximation is a ratio of 5th degree polynomials, and for $4.0 < x$ the rational approximation is a ratio of 3rd degree polynomials.

The approximation of the error function by ERF = 1.0*SIGN (x), for large $|x|$ is dictated by the fact that erf(x) = 1.0 to within machine precision. The approximation of the complementary error function by ERFC(x) = 0 is dictated by the fact that for large values of x, erfc(x) is 0 to within machine exponent range.

5. Space, Time, and Accuracy Considerations

ERF and ERFC provide at least 11 significant figures throughout the entire range of arguments. The amount of time required for a single call to ERF or ERFC is dependent on the argument range. For specific machine-dependent details see Appendix B.

6. Testing Methods

6.1 General

Since the error function and the complmentary error function are both real-valued functions of a single real variable, the testing of these routines consists mainly of determining the number of significant digits throughout the argument range.

The methods available for determining the number of correct significant digits include comparing computed values with published tables, checking computed values for consistency with well-known identities, and comparing computed values with values computed by other routines using other approximations.

6.2 Kinds of Tests Used

The basic test used in certifying ERF and ERFC was a comparison of values computed using ERF and ERFC with entries in a 15-digit table of the error function contained in Reference 2. This table lists values in the range X = 0(0.0001)5.6. For smaller arguments a comparison was made with the Taylor series expansion and with Chebyshev approximations given in Reference 3. The routines were also checked for continuity across the breaks in the argument range.

A further test was performed by comparing ERF and ERFC against double precision routines employing higher order approximations given by Cody.[1] A search was performed for the maximum relative and absolute errors in the region $-6 \leq x \leq 25$.

6.3 Normal Cases Tested

The basic ranges of arguments tested in ERF and ERFC was -30(.1)30 with approximately 1/4 of these values checked against known table entries. The remaining values were scanned for consistency and in comparison against other known approximations for the error function and complementary error functions as described above. ERF and ERFC were also compared for 10,000 arguments in [-30,30] against a double precision routine with higher order approximations and the largest relative and absolute errors ascertained. ERF and ERFC were also tested for continuity at the breaks in the argument range where the approximations changed. This was done by computing ERF(x) and ERFC(x) for $x = 0.46875 \pm 10^{-k}$ and $x = 4.0 \pm 10^{-k}$ for $k = 1(1)10$. Some of the testing results are presented in Appendix B.

6.4 Difficult Cases Tested

There are no difficult or pathological cases associated with ERF or ERFC.

6.5 Range, Error, and Fault Checks Tested

There are no range, error, or fault checks associated with ERF or ERFC.

7. Remarks

None are required for this report.

8. Certification

This routine was subjected to a wide variety of tests. The performance of the routine throughout the tests was checked carefully. The nature of the tests, the reliability of the routine, the error analyses conducted, and the observed variation in accuracy are reported in this document. While it is believed that the facts recorded and the judgments expressed regarding accuracy and reliability are strong indications of the general quality and validity of the routine, the tests should not be considered to be exhaustive. The use of this routine outside of the stated range of application or in violation of stated restrictions may produce unspecified results. The statements made in this document are intended to apply only to those versions of the indicated routine which are released by the Sandia Laboratories Mathematical Program Library Project.

The author selected and implemented the methods, performed the indicated tests, and prepared this document.

APPENDIX A

Listings of ERF and ERFC

```
      FUNCTION ERFC(XX)
C
C     SANDIA MATHEMATICAL PROGRAM LIBRARY
C     MATHEMATICAL COMPUTING SERVICES DIVISION 9422
C     SANDIA LABORATORIFS
C     P. O. BOX 5800
C     ALBUQUERQUE, NEW MFXICO  87115
C
C     WRITTEN BY J.E. VOGEL FROM APPROXIMATIONS DERIVED BY W.J. CODY .
C
C     CONTROL DATA 6600 VERSION
C
C     ABSTRACT
C
C         ERFC(X) COMPUTES 2.0/SQRT(PI) TIMES THE INTEGRAL FROM X TO
C         INFINITY OF EXP(-X**2). THIS IS DONE USING RATIONAL APPROX-
C         IMATIONS.
C
C     DESCRIPTION OF PARAMETERS
C
C         X IS ANY REAL VARIABLE
C
      DIMENSION P1(4),Q1(4),P2(6),Q2(6),P3(4),Q3(4)
      DATA (P1(I),I=1,4)/242.6679552305318,21.97926161829415,6.996383488
     1619136,-3.560984370181539E-02/,
     2(Q1(I),I=1,4)/215.0588758698612,91.16490540451490,15.082797630407
     739,1.0/,
     4(P2(I),I=1,6)/22.898992851659,26.094746956075,14.571898596926,4.26
     5577201070898,.5643716068638l,-6.0858151959688E-06/,
     6(Q2(I),I=1,6)/22.898985749891,51.933570687552,50.273202863803,26.2
     7788795758761,7.5688482293618,1.0/,
     8(P3(I),I=1,4)/-1.21308276389978E-2,-.1199039552681460,-.2439110294
     9988626,-3.24319519277746E-2/,
     1(Q3(I),I=1,4)/4.30026643452770E-02,.489552441961437,1.437712279371
     1218,1.0/
      DATA SQPI/.564189583547756/
C
      X=ABS(XX)
      X2=X*X
      IF(XX.LT.-6.0)GO TO 320
      IF(X.GT.25.8)GO TO 330
      IF(X.GT.4.0)GO TO 300
      IF(X.GT..46875)GO TO 200
      A= X*(P1(1)+X2*(P1(2)+X2*(P1(3)+X2*P1(4))))
      A=A/(Q1(1)+X2*(Q1(2)+X2*(Q1(3)+X2*Q1(4))))
      IF(XX.LT.0.)A=-A
      ERFC=1.0-A
      GO TO 400
  200 A=EXP(-X2)*(P2(1)+X*(P2(2)+X*(P2(3)+X*(P2(4)+X*(P2(5)+X*P2(6))))))
      A=A/(Q2(1)+X*(Q2(2)+X*(Q2(3)+X*(Q2(4)+X*(Q2(5)+X*Q2(6))))))
      IF(XX.LE.0.0)GO TO 210
      ERFC=A
      GO TO 400
  210 ERFC=2.0-A


      GO TO 400
  300 XI2=1./X2
      R=XI2*(P3(1)+XI2*(P3(2)+XI2*(P3(3)+XI2*P3(4))))/(Q3(1)+XI2*(Q3(2)+
     1XI2*(Q3(3)+XI2*Q3(4))))
      A=EXP(-X2)*(SQPI+R)/X
      IF(XX.LT.0.0) GO TO 310
      ERFC=A
      GO TO 400
  310 ERFC=2.0-A
      GO TO 400
  320 ERFC=2.0
      GO TO 400
  330 ERFC=0.0
  400 RETURN
      END
```

```fortran
      FUNCTION ERF(XX)
C
C     SANDIA MATHEMATICAL PROGRAM LIBRARY
C     MATHEMATICAL COMPUTING SERVICES DIVISION 9422
C     SANDIA LABORATORIES
C     P. O. BOX 5800
C     ALBUQUERQUE, NEW MEXICO  87115
C
C     WRITTEN BY J.E. VOGEL FROM APPROXIMATIONS DERIVED BY W.J. CODY .
C
C     CONTROL DATA 6600 VERSION
C
C     ABSTRACT
C
C          ERF(X) COMPUTES 2.0/SQRT(PI) TIMES THE INTEGRAL FROM 0 TO X
C          OF EXP(-X**2). THIS IS DONE USING RATIONAL APPROXIMATIONS.
C
C     DESCRIPTION OF PARAMETERS
C
C          X IS ANY REAL VARIABLE
C
      DIMENSION P1(4),Q1(4),P2(6),Q2(6),P3(4),Q3(4)
      DATA (P1(I),I=1,4)/242.6679552305318,21.97926161829415,6.996383488
     1619136,-3.560984370181539E-02/,
     2(Q1(I),I=1,4)/215.0588758698612,91.1649054045149O,15.0827976304077
     39,1.0/,
     4(P2(I),I=1,6)/22.898992851659,26.094746956075,14.571898596926,4.26
     577201070898,.5643716O686381,-6.0858151959688E-06/,
     6(Q2(I),I=1,6)/22.898985749891,51.933570687552,50.273202863803,26.2
     788795758761,7.5688482293618,1.0/,
     8(P3(I),I=1,4)/-1.21308276389978E-2,-.1199039552681460,-.2439110294
     9886264,-3.24319519277746E-2/,
     1(Q3(I),I=1,4)/4.30026643452770E-02,.489552441961437,1.437712279371
     218,1.0/
      DATA SQPI/.564189583547756/
C
      X=ABS(XX)
      IF(X.GT.6.0)GO TO 320
      X2=X*X
      IF(X.GT.4.0)GO TO 300
      IF(X.GT..46875)GO TO 200
      A= X*(P1(1)+X2*(P1(2)+X2*(P1(3)+X2*P1(4))))
      A=A/(Q1(1)+X2*(Q1(2)+X2*(Q1(3)+X2*Q1(4))))
      IF(XX.LT.0.)A=-A
      FRF=A
      GO TO 400
  200 A=EXP(-X2)*(P2(1)+X*(P2(2)+X*(P2(3)+X*(P2(4)+X*(P2(5)+X*P2(6));))))
      A=A/(Q2(1)+X*(Q2(2)+X*(Q2(3)+X*(Q2(4)+X*(Q2(5)+X*Q2(6))))))
      ERF=SIGN((1.0-A),XX)
      GO TO 400
  300 XI2=1./X2
      R=XI2*(P3(1)+XI2*(P3(2)+XI2*(P3(3)+XI2*P3(4))))/(Q3(1)+XI2*(Q3(2)+
     1XI2*(Q3(3)+XI2*Q3(4))))
      A=EXP(-X2)*(SQPI+R)/X

      ERF=SIGN((1.0-A),XX)
      GO TO 400
  320 CONTINUE
      ERF=XX/X
  400 RETURN
      END
```

APPENDIX B

Results of Testing on the CDC 6600

APPENDIX B

Results of Testing on the CDC 6600

This appendix consists of two tables of test results, and timing and space information. The first compares ERF with tabulated data from Reference 2 and the second shows comparison of ERF with a truncated Taylor series for very small arguments, i.e., 10^{-k} for k = 100, 50, 30, 20, 10, 5, and 4.

TABLE B-I

X	ERF(X)	Tabular Value of Error Function
0.0	0.0	0.0
0.001	0.0011283787910	0.001128378790969
0.01	0.0112834155559	0.011283415555850
0.1	0.1124621960184	0.112462916018285
0.5	0.5204998778131	0.520499877813047
1.0	0.8427007929497	0.842700792949715
2.0	0.995322265018885	0.995322265018953
4.0	0.9999999845827	0.999999984582742
5.0	0.9999999999985	0.999999999998463
6.0	1.000000000000000	1.000000000000000

TABLE B-II

k	ERF(X)	Taylor Series
100	1.128379167095-100	1.128379167095-100
50	1.128379167095-50	1.128379167095-50
30	1.128379167095-30	1.128379167095-30
20	1.128379167095-20	1.128379167095-20
10	1.128379167095-10	1.128379167095-10
5	1.128379167058-05	1.128379167058-05
4	1.128379163334-04	1.128379163334-04

An inspection of the results of testing for continuity at breaks in the approximations showed that the routine functions properly in these areas.

No published tables of the complementary error function are known, but comparison of ERFC against other routines and against values of 1.0 - error function has empirically established that ERFC provides at least 11 significant figures.

In the search of 10,000 equally spaced arguments in the interval [-20,20] the maximum relative and absolute errors found in ERF were 4×10^{-12} and 2×10^{-12}, respectively. For ERFC the comparable figures were 2×10^{-12} and 5×10^{-12}.

ERF and ERFC each require about 260_8 storage locations. The time required for execution depends on the argument value and will vary between 7×10^{-5} and 1.1×10^{-4} seconds.

APPENDIX C

Control Cards for Use on the CDC 6600

APPENDIX C

Control Cards for Use on the CDC 6600

ERF and ERFC are maintained in a library file for the convenience of the Control Data 6600 users at Sandia Laboratories, Albuquerque, New Mexico. The name of the file is MATHLIB. Questions concerning the availability of ERF or ERFC on the Control Data 6600 at Sandia Laboratories, Livermore, California, should be directed to the Numerical Applications Division 8321.

One control card, COLLECT, is required for using the mathematical library file. The COLLECT processor operates on one relocatable binary file and from one to six library files. The library files are searched for routines which contain entry points matching external references in the relocatable binary file. Such routines are added to the relocatable binary file.

A complete typical example follows:

```
JOB CARD
ACCOUNT CARD
FUN,S.
COLLECT,LGO,MATHLIB.
REDUCE.
LGO.
7/8/9 punch in column 1
Program
7/8/9 punch in column 1
Data
6/7/8/9 punch in column 1
```

In the above example, external references in LGO are satisfied, if possible, by selectively adding routines to LGO from MATHLIB. Additional information on the COLLECT processor with examples is contained in UR0004/6600.[4]

APPENDIX D

Computation of Related Probability Functions

APPENDIX D

Computation of Related Probability Functions

The error function and complementary error function are closely related to the normal or gaussian, probability functions P(x)*, Q(x), A(x) where

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dt \ ,$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2}\, dt \ ,$$

$$A(x) = \frac{1}{\sqrt{2\pi}} \int_{-x}^{x} e^{-t^2/2}\, dt \ .$$

Recalling that

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2}\, dt \ , \quad \text{and}$$

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2}\, dt \ ,$$

it can easily be seen that P(x), Q(x), and A(x) can be written by a suitable transform of variable in terms of the error function and/or complementary error function.

---

*P(x) is the function which is presently computed by the function subprogram GOFU(X) on the CDC 3600.

When this is done we find that

$$P(x) = \tfrac{1}{2} \text{ erfc } (-x/\sqrt{2})$$

$$Q(x) = \tfrac{1}{2} \text{ erfc } (x/\sqrt{2})$$

$$A(x) = \text{erf } (x/\sqrt{2}) \; .$$

Thus, the routines ERF and ERFC may easily be utilized to compute P(x), Q(x), and A(x). A few simple Fortran statements will yield function subprograms (see below) to compute these functions.

```
FUNCTION P(X)
P = .5 * ERFC (-X/SQRT(2.))
RETURN
END

FUNCTION Q(X)
Q = .5 * ERFC (X/SQRT(2.))
RETURN
END

FUNCTION A(X)
A = ERF (X/SQRT(2.))
RETURN
END
```

REFERENCES

1. W. J. Cody, Rational Chebyshev Approximations for the Error Function, Mathematics of Computation, July 1969.

2. Tables of Error Function, U. S. Dept. of Commerce, National Bureau of Standards, AMS 41, 1954.

3. C. W. Clenshaw, Mathematical Tables, vol. 5, National Physical Laboratory, Her Majesty's Stationery Office, London, 1962.

4. P. A. Lemke, Auxiliary Library Routines (PREP and COLLECT), Sandia Computing Publication UR0004/6600, June 1969.

DISTRIBUTION:

W. C. Myre, 1210
O. M. Stuetzer, 1220
D. M. Olson, 1510
V. E. Blake, Jr., 1520
T. B. Lane, 1540
S. W. Key, 1541
R. L. Peurifoy, Jr., 1550
I. M. Moore, 1610
B. E. Arthur, Jr., 1640
R. R. Prairie, 1643
W. L. Stevens, 1650
C. Winter, 1710
D. R. Morrison, 1720
H. H. Wicke, 1721
R. J. Thompson, 1722
R. G. Clem, 1730
H. D. Sivinski, 1740
A. A. Lieber, 1750
C. B. McCampbell, 2310
K. L. Gillespie, 2320
J. P. Shoup, 2330
M. T. Abegg, 2340
W. C. Kraft, 2440
T. S. Church, 2450
C. M. Tapp, 2610
G. W. Rodgers, 2620
E. G. Franzak, 2630
J. A. Hood, 2650
G. W. Gobeli, 5110
O. E. Jones, 5130
J. E. Schirber, 5150
W. Herrmann, 5160
L. D. Bertholf, 5162
C. H. Karnes, 5165
A. W. Snyder, 5220
C. R. Mehl, 5230
J. H. Renken, 5231
E. H. Beckner, 5240
H. M. Stoller, 5310
J. E. McDonald, Actg., 5320
R. L. Schwoebel, 5330
R. G. Kepler, 5510
W. M. O'Neill, 5520
M. J. Davis, 5530
R. L. Brin, 7250
T. L. Pace, 7260
J. E. Stiegler, 7280
L. E. Hollingsworth, Actg., 7290
E. K. Montoya, 7292
M. McWhirter, 7320
M. Cowan, 7340
F. W. Neilson, 7360
S. A. Moore, 7370
J. R. Meikle, 7410
R. H. Schultz, 7420
J. R. Sublett, 7430
L. J. Heilman, Actg., 7450
J. W. Jones, 7510
W. E. Caldes, 7520
R. J. Dill, 7530
F. F. Eichert, 7610
R. W. Devore, 7620
L. M. Jercinovic, 7630
B. H. Bright, 7650
J. W. Pearce, 8120
J. D. Benton, 8121
D. E. Gregson, 8130
L. E. Davies, 8150
R. A. Baroody, 8160
G. E. Brandvold, 8170
C. S. Selvage, 8180
R. H. Meinken, 8310
J. W. Weihe, 8320
J. N. Rogers, 8321
A. G. Schuknecht, 8322
G. W. Anderson, Jr., 8330
J. L. Wirth, 8340
A. N. Blackwell, 8350
C. D. Broyles, 9110
G. E. Hansche, 9120
H. E. Viney, 9130
J. R. Banister, 9150
M. L. Kramm, 9210
J. C. Eckhart, 9220
H. H. Patterson, 9230
R. E. Hepplewhite, 9240
R. C. Maydew, 9320
W. H. Curry, 9322
E. C. Rightley, 9323
H. R. Vaughn, 9325
D. F. McVey, 9328
K. J. Touryan, 9340
W. C. Scrivner, 9400
L. E. Mahuron, 9410
J. E. Wesbrook, 9411
C. E. Katzenberger, 9412
G. D. Horne, Jr., 9413
E. C. Domme, 9414
J. H. Porter, 9415
J. L. Tischhauser, 9420
A. R. Iacoletti, 9421
D. A. Young, 9422
J. E. Vogel, 9422 (50)
D. K. Robbins, 9424
B. T. Fox, 9427
P. Van Delinder, 9427
For: COSMIC (3)
A. J. Arenholz, 9428
A. J. Clark, Jr., 9510
G. J. Hildebrandt, 9520
L. C. Baldwin, 3412
G. C. McDonald, 3416 (3)
Attn: J. G. Wallace
C. H. Sproul, 3428-2 (65)
Library Division, 8232 (5)