Lightning Phenomenology

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LIGHTNING RETURN-STROKE TRANSMISSION LINE MODEL*

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ABSTRACT

We present a transmission-line model for the lightning return stroke. This model incorporates a model for the corona surrounding the charged lightning channel as well as a model for the evolution of the channel incorporating hydrodynamics, energy losses, joule heating. We find that the model reproduces the "double exponential" shape of the observed field in the radiation zone, without imposing a priori the time behavior of the current pulse. The decrease in field at late times preceeds any decrease in current, and is due to a lengthening of time scale. The current pulse is substantially altered as it progresses up the lightning channel, and the radiation zone signal is not of the same shape as the current pulse at any position or time.

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I. INTRODUCTION

The modeling of a lightning channel is sufficiently difficult to call for a number of simplifying assumptions to make analysis tractable, even if numerical simulations are being used. One popular approximation is to treat the channel as a transmission line (Refs. 1-4). The lightning channel is considered to be a single conductor extending vertically above a conducting ground plane. Often, the line is considered lossless and a current waveform is assumed to propagate down the line (Ref. 1). Antenna theory then gives relatively simple expressions for the resulting electric and magnetic fields (Ref. 5). Price and Pierce (Ref. 2) introduced the refinements of a resistive channel with an initially charged transmission line, as opposed to one with a "signal" current imposed at the ground. Little (Ref. 3) considered in greater detail the line parameters, although these were still independent of time and the uncharged line model was employed. Gardner (Ref. 4) used a complex hydrodynamic code to study the evolution of the channel parameters with the passage of the current, again employing an initially uncharged line.

Natural lightning is considerably more complex than any of these models, or course. The channel branches and is tortuous. The impedance behavior of the point of attachment of the discharge with the ground is complex. The lightning channel may be expected to be surrounded by a corona. This is suggested for example, by the observation that the photographic or luminous diameter (few centimeters to meters) is generally much larger than the diameter of the current carrying channel (millimeters to centimeters), when the latter is inferred from damage to mesh screens (Ref. 6). Golde (Ref. 7), in discussing transmission line models, quotes Schonland as stating:

> "the observations as a whole clearly indicate that the return streamer is engaged in removing charge which has been brought into the air by the leader and is held chiefly on its branches"

Wagner (Ref. 8) also discusses the return stroke as:

> "a wave of positive charge (and current) whose head travels upward, neutralizing, as it progresses, the negative charge laid down by the downward leader."

It therefore would seem to be useful if not essential to model the storage of charge in the corona surrounding the channel, and to treat the evolution of the stroke current as resulting from the flow of this stored charge.

The model to be discussed here incorporates most of the physics discussed above. Channel tortuosity and branching is not explicitly included, although the average coronal parameters could be considered to be determined by such effects in this model. The charge stored on branches could be thought to give rise to an additional average capacitance, for example. Tortuosity increases the microscopic channel length for a given macroscopic channel length. On a scale smaller than the coronal radius, such channel tortuosity will raise the resistance and inductance per unit length. On a larger scale, as it will raise both the inductance and capacitance proportionally, the channel impedance will be unchanged. The wave velocity will be slowed, if judged by observed front progression along the macroscopic channel distances. These various effects can be parametrized by simple multipliers, as done by Strawe (Ref. 9), but we have not done so here.

The corona model is the simple model developed by Baum (Ref. 10) for lightning channels. If dissipative effects are neglected, it may be treated analytically (Ref. 11). We will see that dissipative processes have a significant effect. It is interesting to note that coronal effects on a lightning channel will be opposite those expected on a power line (Ref. 12). In the power line case, the corona builds up as the wave propagates, drawing energy and eroding the wave front. On a lightning channel, it is the corona that is being tapped and consequently, there is a tendency for the wave front to steepen. This is opposed by the effects of channel resistance. Those effects, for reasonable channel parameters, are shown in this study to dominate. At present, the understanding of coronal phenomena is rudimentary, and our corona model is very simple. We model the evolution of the lightning channel with the "two-zone" model (Ref. 13) developed previously. This model is an elaboration of a one-zone model developed by Strawe (Ref. 9) and based upon the model of Braginskii (Ref. 14).

With such a model we are able to eliminate many ad hoc assumptions previously needed in transmission line models. Some assumptions must still be made, however these are all grounded in physics or observed lightning properties. The one parameter not well grounded in physics is the "radius of current return", which in effect sets the wave velocity. This can of course be chosen to give the desired wave velocity, which is in effect what all previous models have done. We discuss in the conclusion section methods of eliminating this arbitrariness.

The principal conclusion of this work is that realistic models can be obtained. Further, the current pulse on the resistive transmission line model used here does not have the same shape as the far zone radiation field signal, as it would have in an ideal model (Ref. 5). This is because the pulse on the realistic lightning channel does not propagate without distortion. The "exponential tail" of the signal is due to a decrease in the time derivative of the current because of an increase in the time scale with propagation, not a decrease in the current.

II. MODEL DESCRIPTION

A segment of the transmission line is shown in Figure 1. The series resistance per unit length R represents the the finite resistance of the channel. The joule heating of the lightning channel results in the observed optical emission of the channel during the passage of the stroke current. This resistance is computed using the channel model (Ref. 13) mentioned above. The model computes the properties of two zones: an inner, conductive plasma core and a surrounding shell which has been shocked into a higher density and temperature than the ambient air. The shock wave defines the boundary between this shell and the ambient. This model differs from the one-zone models (Refs. 9, 14) which assume the shock wave does not separate from the edge of the hot channel. Such models overestimate the channel radius and consequently underestimate the resistance per unit length of the channel. The two-zone model in additions incorporates improved treatments of the air equation of state, radiative losses, and the shock propagation.

The inductance L and capacitance C per unit length of the channel are standard, and require an assumed "return current radius". This corresponds in

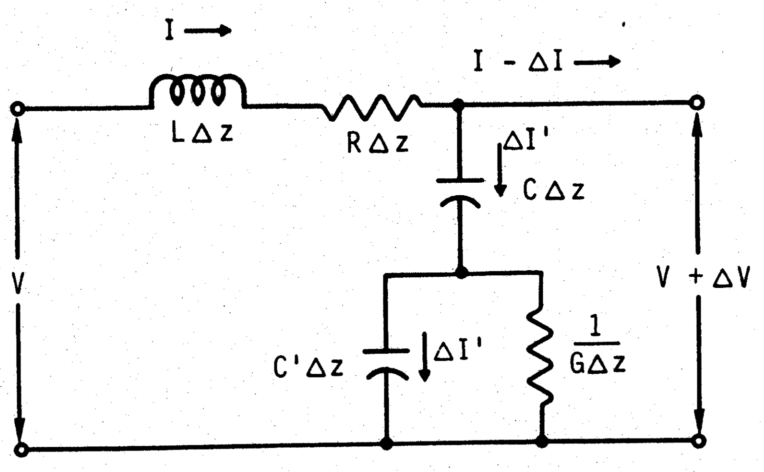

Figure 1. An element in the transmission line model for the lightning channel.

other models to a choice of wave velocity $v = 1/\sqrt{(LC)}$, or equivalently characteristic impedance $\sqrt{(L/C)}$. For the present model we take this radius as a fixed parameter. Little (Ref. 4) went to some effort to compute effective values for L and C using electrostatic theory, but it is not clear that this resulted in a significantly improved model. An estimate of this radius can be made using antenna theory (Ref. 14). One finds that the effective characteristic impedance, and hence this radius and L and C, is a function of frequency. For a chosen "dominant" frequency, one can choose an appropriate return current radius. This is discussed below. At present, then, following Baum (Ref. 14), we have

$$L = 2 \times 10^{-7} \ln(r_{return}/r_{channel})$$

$$C = 2 \pi \varepsilon / \ln (r_{return}/r_{corona})$$

R = determined by channel radius, temperature model

The corona parameters are C' and G, a capacitance and conductance per unit length. The former measures the ability of the corona to store charge; the latter estimates the conductivity of the weakly ionized coronal volume, and limits the rate at which charge may be drawn out of the corona. These are related simply to the corona's radius; the conductivity of the corona is not predicted by theory at present, and in simulations to date we have ignored this term, setting G to a very small value. The coronal radius is calculated, following Baum (Ref. 10), with the simple approximation that it is equal to that radius at which the radial electric field equals the static breakdown stress of 3 MV/m. This radius is calculated by a Newton-Raphson iteration at each point along the transmission line, given the channel voltage and radius. Thus

$$C' = 2 \pi \varepsilon / \ln (r_{corona}/r_{channel})$$

$$G = 0$$

and r_{corona} is determined from the solution to

$$V/E = r_{corona} \ln (r_{return}/r_{corona})$$

where V is the line voltage and E is the static breakdown field of 3 MV/m. The equations are then

$$\frac{\partial I}{\partial t} = - \frac{RI}{L} - \frac{1}{L}\frac{\partial V}{\partial z}$$

$$\frac{\partial V}{\partial t} = - \frac{1}{C}\frac{\partial I}{\partial z} - \frac{1}{C'}\frac{\partial I}{\partial z}$$

$$\frac{\partial I'}{\partial t} = \frac{\partial I}{\partial t} - \frac{GI'}{C'}$$

The boundary conditions are fixed impedances at cloud and ground terminations. The initial conditions are zero current flow everywhere and voltage initialized to an initial value along the line, down to an "initiation point" above the ground (typically, two zones) below which the voltage decreases linearly to zero. This smooth decrease was found to prevent initial transients in the numerical solution. We have used a ground impedance of 100 Ω and a cloud termination of an effectively infinite amount. These may easily be varied, of course, to account for better models of attachment and initiation, to study the role of surge impedance of towers, etc. on stroke evolution, or to model possible cloud breakdown phenomena.

The finite-difference equations are similar to those of Gardner (Ref. 4), which are similar to those of Richtmyer and Morton (Ref. 15). We have generalized the algorithm to allow fully implicit instead of time-centered implicit calculations, for increased stability. We found that this was desirable if the resistance per unit length R were small or zero.

III. RESULTS

A vertical channel 1.5 km high was used. We studied in detail cases with initial radii of 0.001 and 0.01 m (1 mm and 1 cm), but experimented as well with channels of varying radius (0.001 m at ground level increasing linearly to 0.01 m at cloud height, simulating the fact that the older portions of the channel had more time to expand). Twenty-five zones in the vertical, each 60 m, were used. The initial coronal radius was found to be approximately 44 cm for this configuration. Figures 2-8 plot results for the two uniform diameter cases for variables of diagnostic interest, including currents, radiated power ("brightness") and the vertical electric field at 50 km distance. This was calculated in the radiation zone (following Ref. 5, neglecting retardation) and is shown in Figure 3. Note the decrease in signal at early times well before signal reflection at cloud height (~8 μs). This is due to the increase in time scale, which is clearly seen in Figure 2b. The front spreads as it advances, due to the finite resistance. Similar behavior was noted by Strawe (Ref. 9), who did not calculate radiated fields and did not model the corona. It is clear that any tendency of the corona to cause the front to steepen is more than overcome by the finite channel resistance, for the parameters employed. The sharp drop in signal from 7 to 9 μs is due to the wavefront reflection from the top of the channel, resulting in a change of propagation direction and a consequent change of signal sign. The computer plots show a "ringing" of the channel (small amplitude, high frequency "noise"). This may be shown by varying the time step and length increment to be due to the discretization of the transmission line equations. This does not mean that they are completely unphysical, however, since a real lightning channel is not a uniform vertical channel. Its branches would act as "lumped" discrete capacitors loading the main channel, and such ringing (as seen in observations) might have an origin similiar in physical basis as the "fuzz" on the computer plots for radiated signal.

Figure 4 shows the radiated signal for a calculation in which $r_{channel}$ = 0.01 m (1 cm) initially. Such a channel has, initially a faster rise and greater peak vertical radiated E field. This quickly decays, however, down to a field typical of the smaller channel. The larger channel diameter case might be more similar to a subsequent return stroke and the smaller diameter channel case to a first return stroke. However, the channel conditions in both cases differ by more than merely the channel radius. The subsequent returns strokes would appear from observations to charge up to a lower voltage before the current wave is launched, since the typical currents are lower. The faster rise typically observed is in accord with the probability that the channel is less resistive, due to larger diameter and possibly higher initial temperature and hence ionization and conductivity. The differences between these two simulations is analagous to observed differences between first and subsequent return strokes. Figures 27-6 and 27-7 of Anderson and Sakshaug (Ref. 16) (based on work of Kroninger (Ref. 17) and Berger (Ref. 18)) show current waveshapes for first and subsequent return strokes. The first return strokes are more like Figure 4, the subsequent strokes more like Figure 6. See also References 19-21 for both current and radiation zone electric field plots.

The results of this paper differ somewhat from Price and Pierce (Ref. 2); because of their desire for an analytic solution, they used initial conditions which resulted in the current waveform having an infinite derivative at the leading edge. This is unphysical and neglects the finite impedance at the ground, which would impose an finite L/R time scale on current rise.

Figures 7 and 8 show the radiated power, and therefore give some measure of the optical brightness, of selected points along the channel.

IV. CONCLUSIONS AND DIRECTIONS FOR FURTHER WORK

On the basis of transmission line models, it has generally been assumed that the time dependence of the current along a lightning channel at any point was the same as the time dependence of the vertical electric field in the radiation zone (e.g, Eq. 20 in Ref. 5, which holds until the current pulse reaches the top of the channel). This has lead to the general use of a "double exponential current pulse", i.e. a current of the form $I(t) = i(\exp(-At) - \exp(-Bt))$. This would seem to conflict with direct current measurements by Berger (Ref. 16). The model presented here resolves such problems by demonstrating that a resistive channel accounts for a double exponential radiated signal from a current monotonically increasing at any point in the channel at corresponding (retarded) times. The resistance used was not imposed ad hoc but followed from the channel radius and temperature, which were calculated accounting for hydrodynamic expansion, radiative losses, joule heating, etc. the model discussed here offers, for the first time, the possibility of predicting the form of I(t) instead of imposing it.

There are nonetheless a number of unsatisfying features of the model which suggest directions for future research. As discussed in Reference 11, the model for the corona is crude and its agreement with experiments is questionable. We would argue a more complete, time-dependent model for corona evolution is desirable.

The only ad hoc parameter in the model is the "return current radius" r_{return}. This determines L and C and through them the wave speed and surge impedance of the channel. Antenna theory (Ref. 22) tells us that the impedance is a function of frequency :

$$Z = \sqrt{(L/C)} = 60\ [\ell n(\lambda/2\ \pi\ a) + 0.116 + Ci(bH) - \sin bH/bH]$$

where H is the channel height, a the channel radius, wavelength $\lambda = c/\nu$ with c the speed of light and ν the frequency, $b = 2\pi/\lambda$ is the wavenumber, and Ci(x) is the Fresnel cosine integral. To a good approximation, $L = 2 \times 10^{-7}$ $\ell n(2h/a)$ where h is the smallest of: H, $\lambda/4\pi$, d/2 where d is a coaxial outer return radius which for a lightning stroke may be taken as many tens of kilometers and consequently neglected. For the highest frequencies, which for a 2 km high channel means above 24 kHz or for time scales shorter than 42 μs, r_{return} is effectively $\lambda/4\pi$. This implies that the lightning channel antenna would be dispersive, with the higher frequencies propagating more slowly along the channel, as compared to a conventional transmission line. One could transform the transmission line equations into frequency domain, with I and V functions of frequency rather than time. The broad bandwidth required (from time scales of nanoseconds to milliseconds) would suggest that uniform intervals in frequency space would be prohibitively expensive. However, the use of nonuniform spacing would mean that the fast Fourier transform could not be used. If a transform to time domain is only done once, at the "end" of the problem, this might not be prohibitive. We would have to forgo studying the evolution of the channel hydrodynamically, however, in order to do this.

A more satisfying but more complicated solution is to use a fully two-dimensional (cylindrically symmetric) field solver. Such codes have been developed for plasma simulation purposes. Their adoption to the lightning channel therefore appears feasible. Such codes would require significantly more computer resources than the one-dimensional transmission line model. Such an expenditure might be necessary, however, if a significant advance is to be made. We are developing a formulation which allows mutual interaction of all the elements of the channel, and hope to be able to apply it soon.

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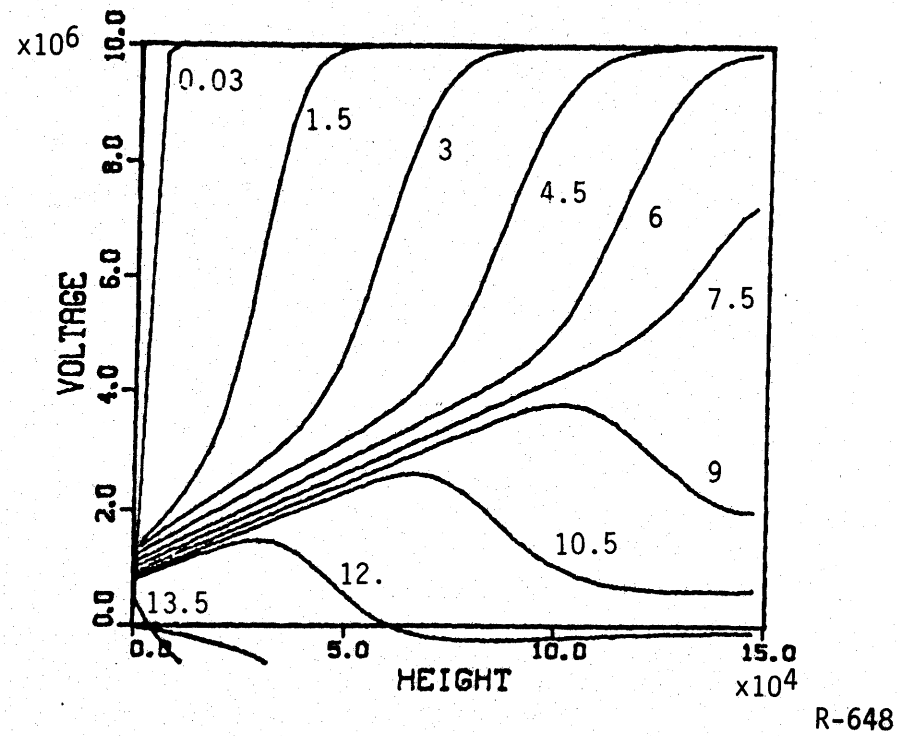

Figure 2a. Voltage versus position at times shown in microseconds, 0.001 m initial radius.

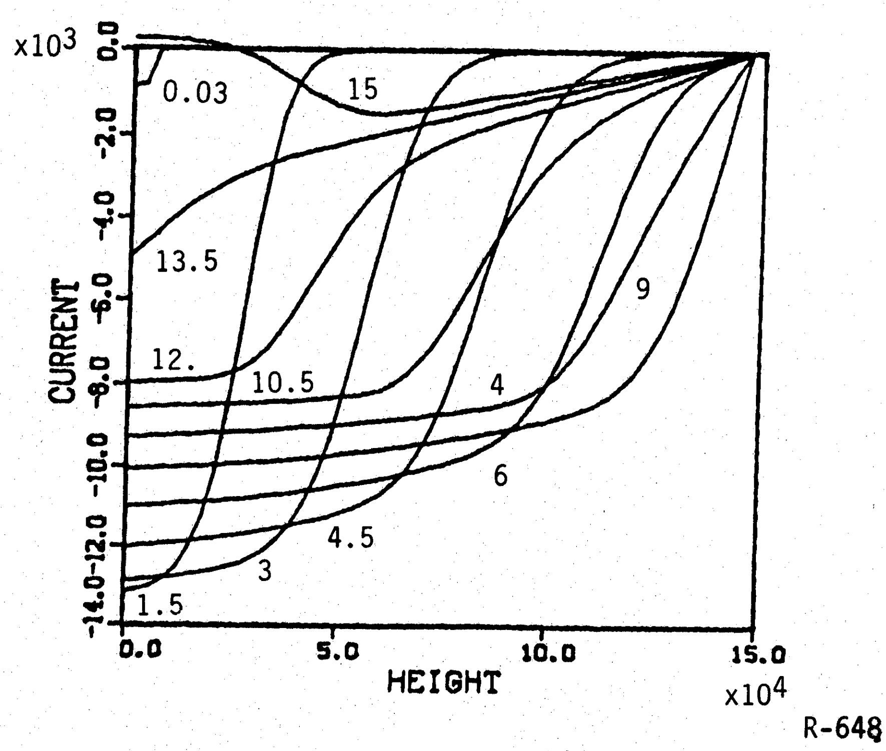

Figure 2b. Current versus position at times shown in microseconds, 0.001 m initial radius.

Figure 2. Results for simulation of return stroke on lightning channel.

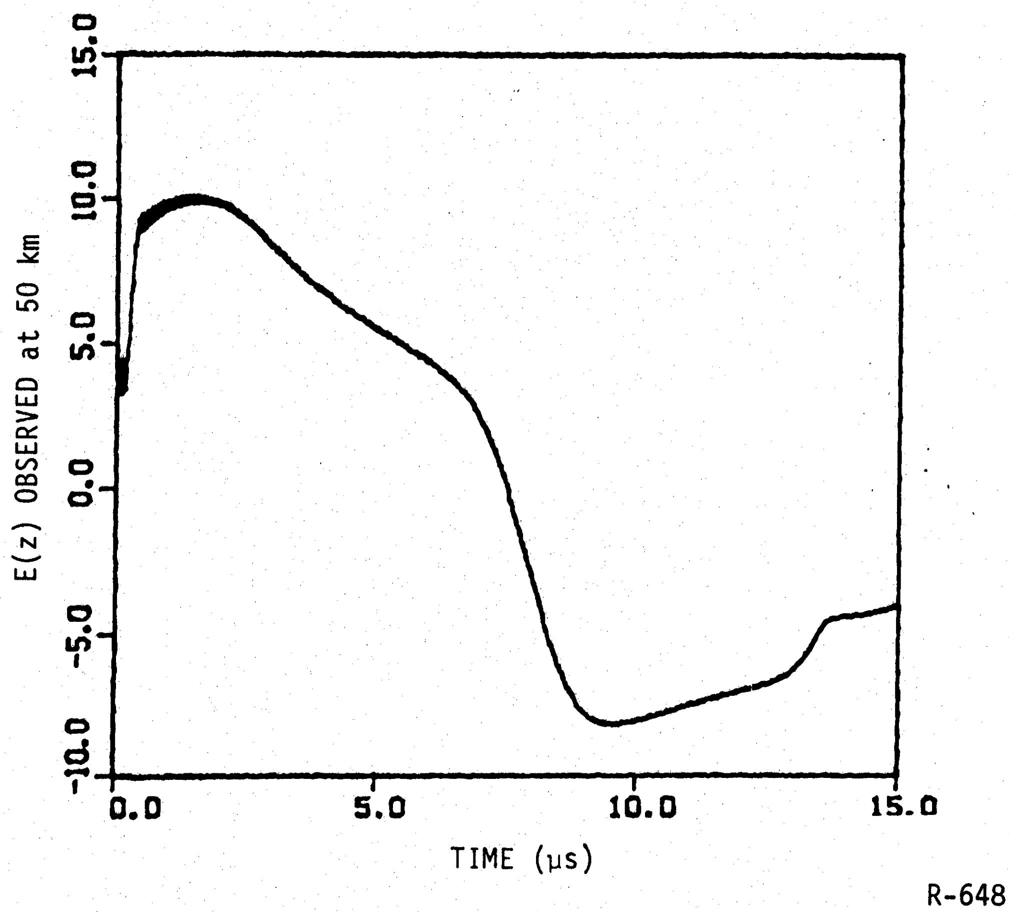

Figure 3. Vertical electric field at 50 km distant observation point on the ground plane, computed using the radiation zone approximation, for the case with 0.001 m initial channel radius.

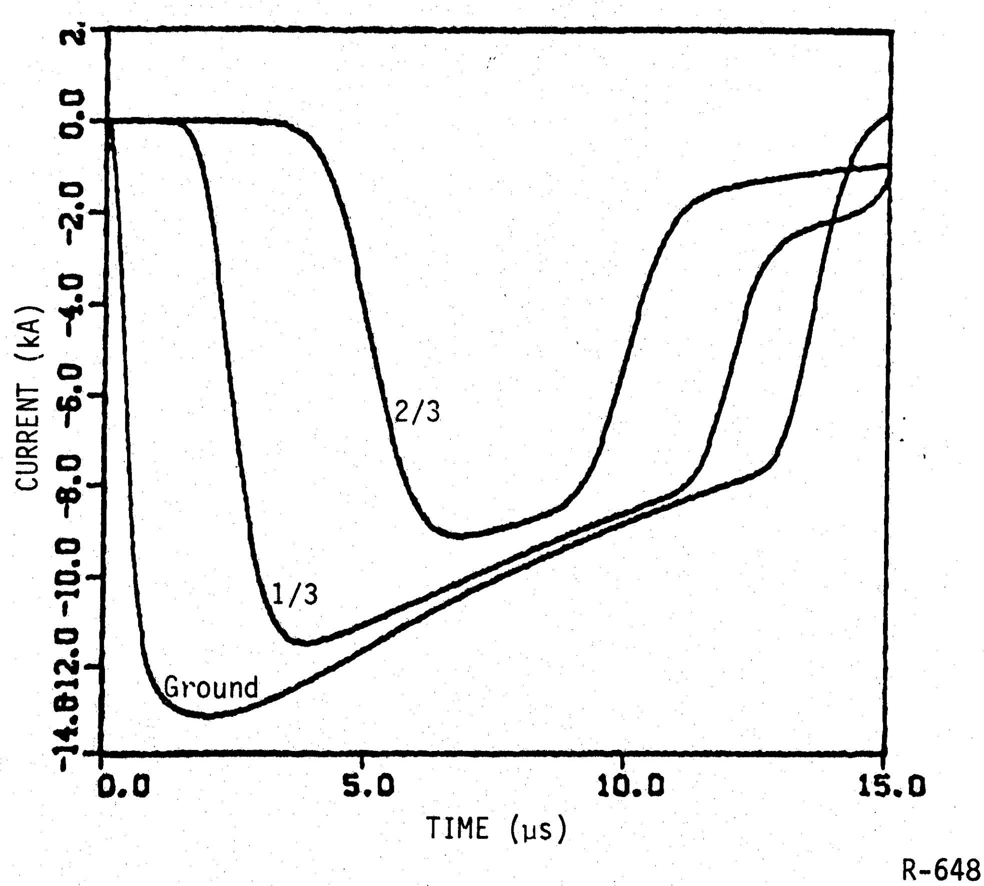

Figure 4. Current in the lightning channel for the case with initial channel radius of 0.001 m, at ground level, 1/3, and 2/3 of the total height above the ground.

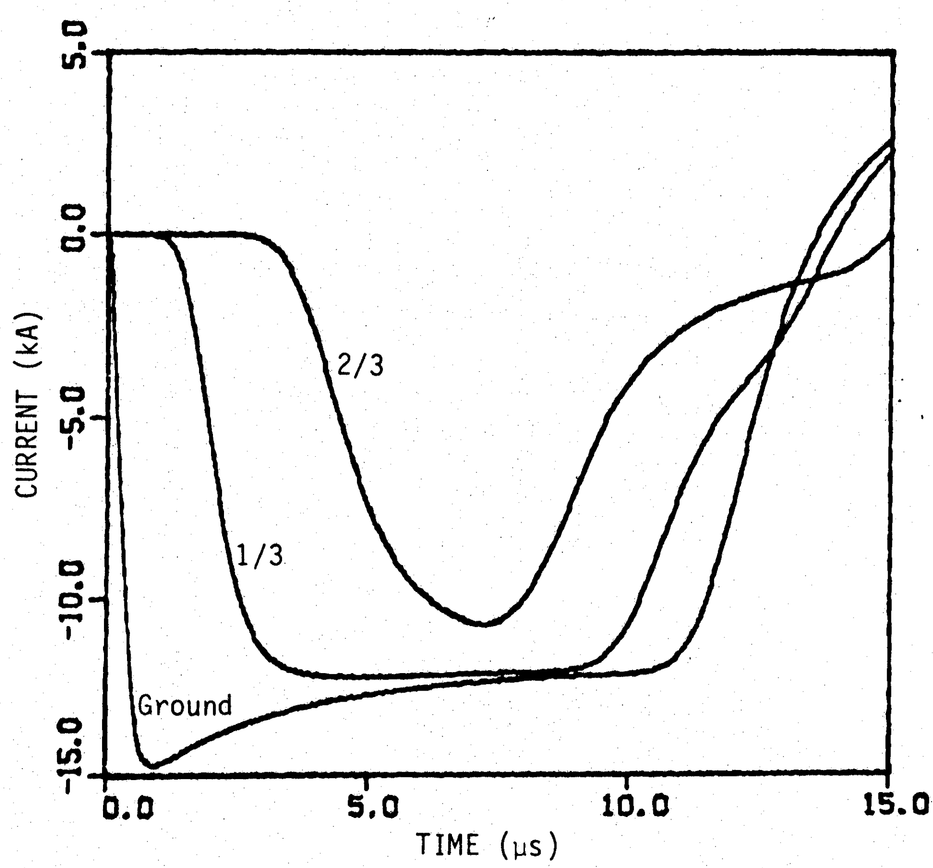

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Figure 5. Vertical electric field at 50 km distant observation point on the ground plane, computed using the radiation zone approximation, for the case with the channel initially 0.01 m radius.

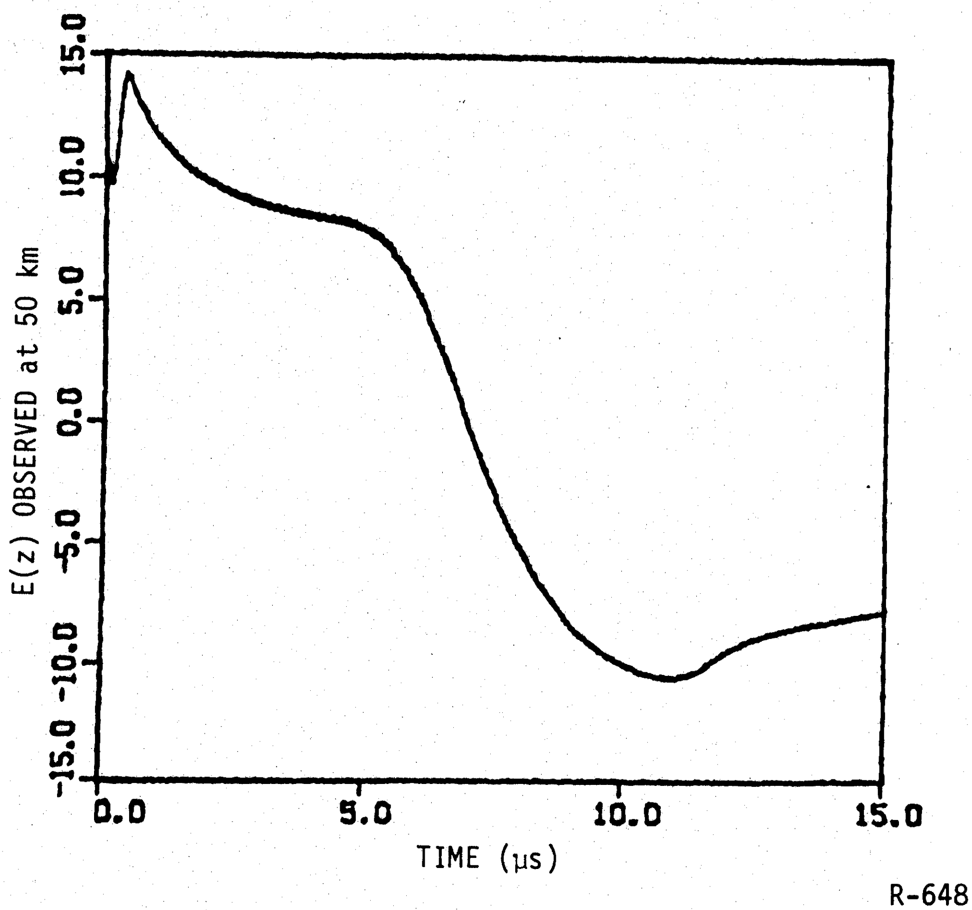

Figure 6. Current in the lightning channel for the case with initial channel radius of 0.01 m, at ground level, 1/3, and 2/3 of the total height above the ground.

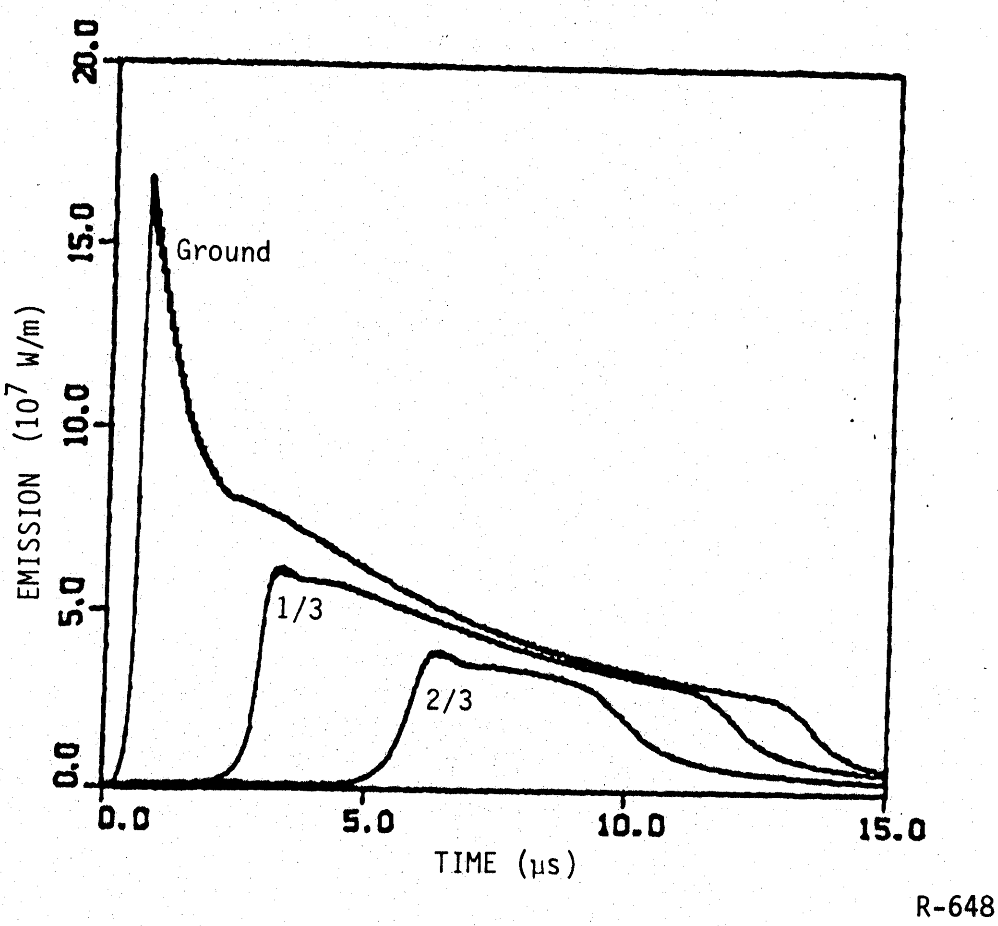

Figure 7. Radiant emission (W/m) from the lightning channel for the case with initial channel radius of 0.001 m, at ground level, 1/3 and 2/3 of the total height above the ground.

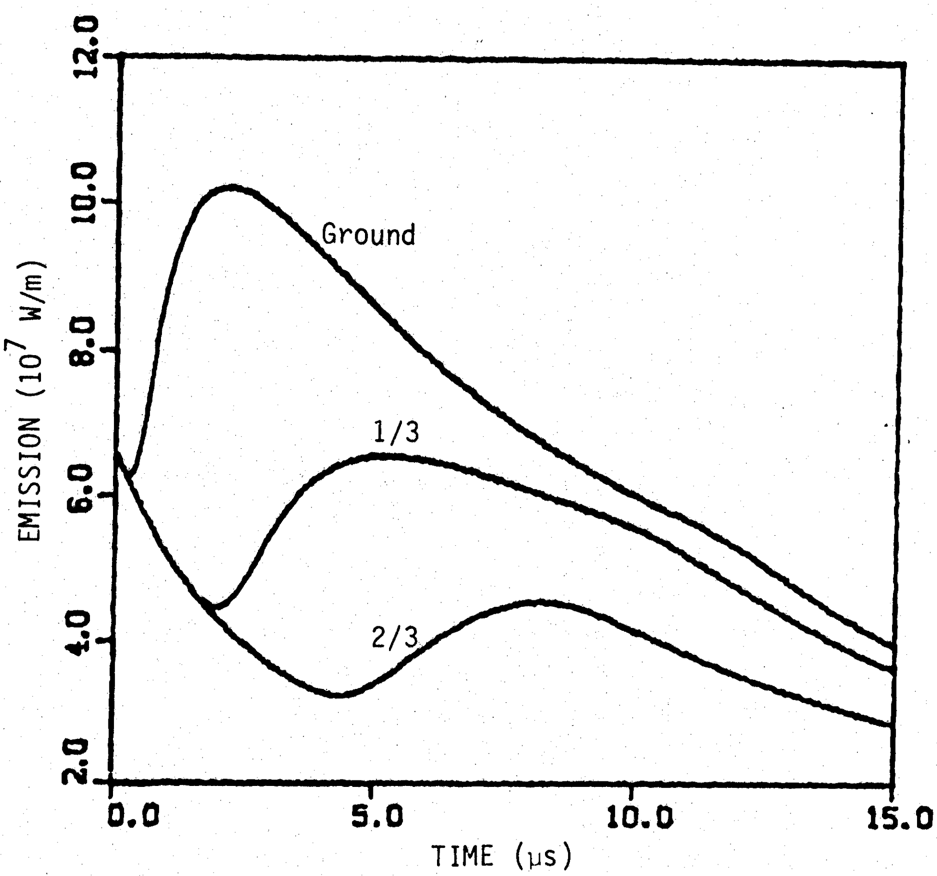

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Figure 8. Radiant emission (W/m) from the lightning channel for the case with initial channel radius of 0.01 m, at ground level.