LIGHTNING PHENOMENOLOGY NOTES

NOTE 5

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AN INTRODUCTION TO LEADER TIP MODELING

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ABSTRACT

Frequently in the lightning literature the upper bound on the rate of rise of lightning current of the order of 10^{11} A/s or a few times this appears. There is currently no theoretical explanation for this limit or the probability distribution around this maximum. In this note we present a simple theory which establishes plausibility of this limit in the case of a leader tip. We also compare the results of the basic theory to more complex treatments applied to nuclear lightning.

CONTENTS

I. INTRODUCTION

A number of investigations have reported values of the maximum rate of rise of lightning current as being around 10^{11} A/s. In a summary of lightning parameters derived from measurements of strikes to towers. Berger et al. (Ref. 1) find that for negative subsequent strokes (their largest values) dI_{max}/dt exceeds 0.12×10^{11} A/s on 95 percent of the return strokes measured, 0.4×10^{11} A/s on 50 percent of the strokes and 1.2×10^{11} A/s on 5 percent of the strokes. Ogawa (Ref. 2) presents a summary of lightning parameters in which $\dot{I}$ is said to exceed 10^{11} A/s on 2 percent of the return strokes measured. The largest rate of rise of lightning current observed in the very recent airborne measurements in the NASA F-106B program is 10^{10} A/s (Ref. 3). C. E. Baum, et al. (Ref. 4) has reported maximum values of the rate of rise for the current of about 10^{11} A/s for a variety of current levels, including leader processes which have characteristic times of 30 ns. For this maximum to exist for processes spanning more than an order of magnitude in current implies that there may be a particular mechanism that limits the rate of rise of current.

In this note we will describe an argument using simple physics of the leader tip which supports the value of about 10^{11} A/s as a maximum plausible value for the rate of rise of the current. The results of this simple model will be compared with the detailed model (Ref. 5) results for the nuclear lightning application.

II. THE LEADER TIP MODEL

Consider a leader propagating into virgin air with a conical shaped tip, as in Figure 1.

Note the two coordinates z and z' related by

$$z' = z + vt$$

since we are assuming a leader tip of constant conical shape propagating at a constant velocity v in the negative z direction. In terms of z' the shape is assumed independent of time. Note cylindrical coordinates (Ψ, ϕ, z) or (Ψ, ϕ, z).

For a distance z' back from the tip, there is an equivalent, charge per unit length Q'(z') which results in an electric field which is approximated by the breakdown field E_b, at radius $\Psi(z')$. That charge per unit length, ignoring finite length and tip effects, is

$$Q'(z') = 2\pi\ \Psi(z')\ \varepsilon_o E_b \frac{1}{\cos(\alpha)}\ u(z')$$

$$u(z') = \begin{cases} 1 \text{ for } z' > 0 \\ 0 \text{ for } z' < 0 \end{cases}$$

This value for the charge is derived using Gauss's theorem. Improving this model by a rounded tip would, however, require going to a distributed charge distribution to avoid a singularity in dI/dt.

The total current passing the observer at some z as the tip propagates is the time derivative of the total charge which has passed the observer.

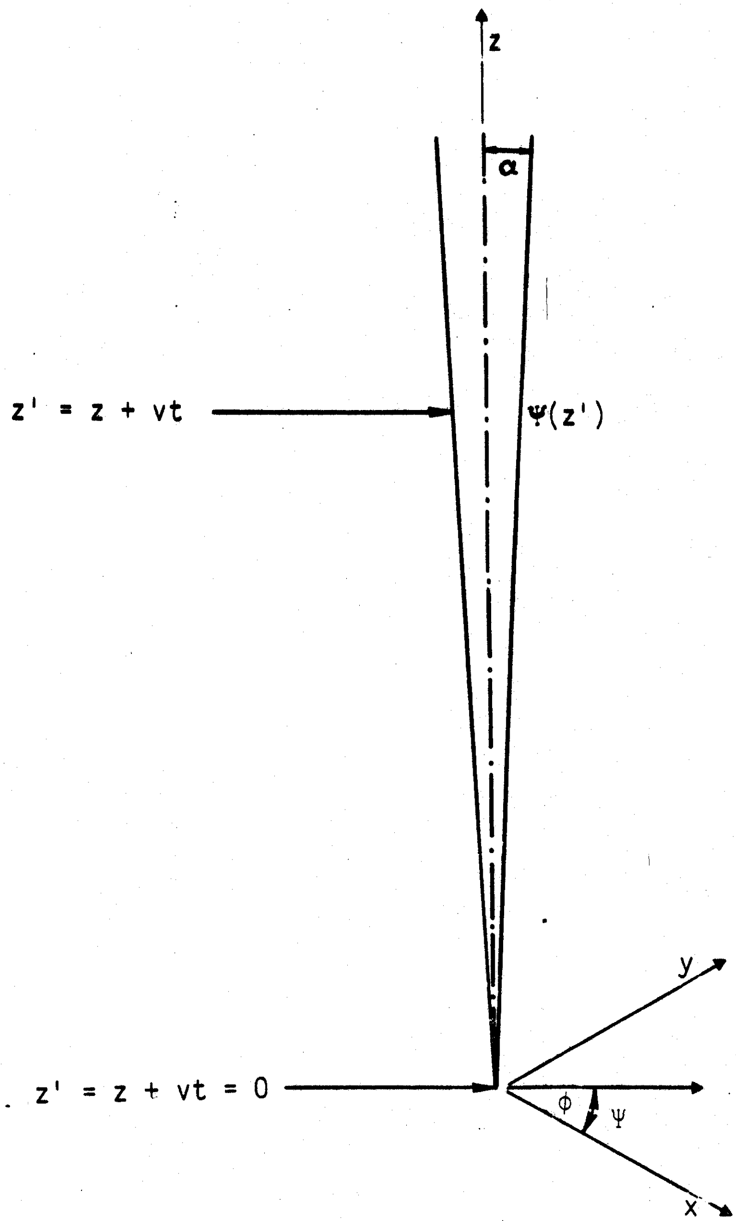

Figure 1. Triangular tip for leader moving at velocity v.

$$I = -\frac{\partial}{\partial t} \int_{-vt}^{z} Q'(z')\, dz' \qquad (2)$$

where I is taken as positive in the +z direction.

For a coordinate system moving with the tip, the radius of the cone is related to distance along the z' axis, for the triangular wavefront by

$$\Psi(z') = \frac{\Psi_o z'}{z_o} = \tan(\alpha)\, z' \qquad (3)$$

where Ψ_o/z_o is a measure of the steepness of the conical wavefront and will be treated as a parameter. Equation 2 may now be integrated

$$I = -\frac{\partial}{\partial t} \int_{o}^{z'} Q'(z'')\, dz'' \qquad (4)$$

$$I = \frac{\partial}{\partial t} \left[2\pi \frac{\Psi_o}{z_o} \varepsilon_o E_b \frac{z'^2}{2} u(z') \right] \qquad (5)$$

The time derivative $\partial(z'^2)/\partial t$ is

$$\frac{\partial}{\partial t}(z'^2) \Big|_{z\ \mathrm{const}} u(z') = 2z'\, u(z') \frac{\partial z'}{\partial t} \Big|_{z\ \mathrm{const}} = 2[z + vt] \frac{\partial}{\partial t} [z + vt] \Big|_{z\ \mathrm{const}}$$

$$= 2\, v[z + vt]\, u(z + vt) \qquad (6)$$

or

$$\dot{I} = 2\pi \frac{\Psi_o}{z_o} \varepsilon_o E_b v[z + vt] \qquad (7)$$

Finally the parameter of interest $\dot{I}$ is

$$\dot{I} \equiv \frac{\partial I}{\partial t}\bigg|_{z\ \text{const}} = 2\pi \frac{\Psi_o}{z_o} \varepsilon_o E_b v^2 u(z + vt) \tag{8}$$

A typical maximum velocity of propagation of the current pulse for a return stroke or leader is about c/3 (Refs. 6, 7). The uniform breakdown field for air is approximately 2×10^6 V/m. When those values are substituted in Equation 8 we have

$$\dot{I} = 1.1 \times 10^{12} \frac{\Psi_o}{z_o} u(z + vt) \text{ A/s} \tag{9}$$

A rate of rise of the channel current which is 10^{11} A/s requires a 5.7 degree tip angle which is sharp and not inconsistent with the detailed tip calculations for the nuclear lightning environment in Reference 5. Requiring an $\dot{I}$ of 10^{12}, however, requires a very blunt tip with about a 45 degree half cone angle. Obviously, better calculations with better geometry and physics are required but this calculation indicates that we are headed in a consistent direction.

As part of the study of nuclear lightning a detailed calculation of the boundary at which $E = E_b$ around a thin conducting filament for a smaller ambient electric field was carried out (Ref. 5). This calculation assumed a stationary channel so that $\dot{I}$ as calculated in this note cannot be easily calculated with the complex geometry. The shape of that boundary is somewhat complex, but generally has an ice cream cone like shape. Figure 2 is a composite of the cone in Figure 1 and the complex shape derived in Reference 5. The ratio of ambient field to breakdown field was assumed for convenience to be 0.02 for the nuclear lightning. If we consider the first 30 ns to 10 ns of our leader tip, and demand that we transition to the cylindrical corona model by that line, then a velocity of c/3 implies that these times correspond to the first 3 to 10 m of leader. At this kind of distance from the tip the nuclear lightning has a radius of about 1 m.

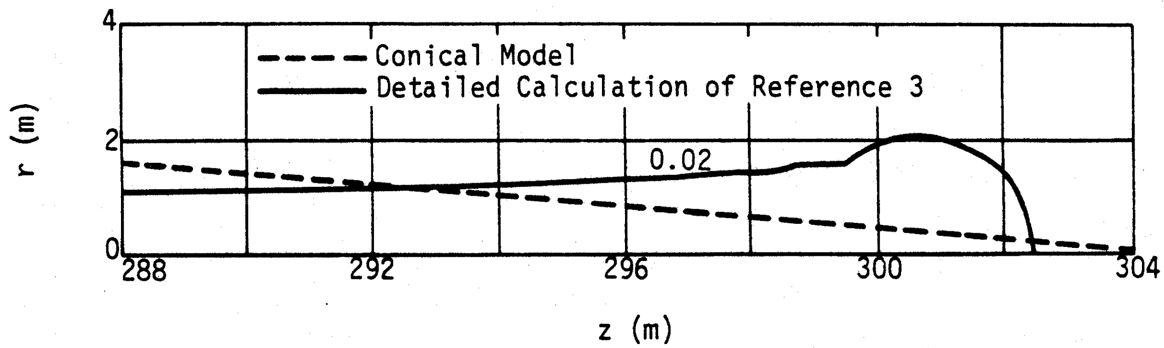

Figure 2. Comparison of the predicted tip geometry for both the conical model of this note and the more detailed calculations of Reference 3.

Furthermore Reference 6 obtains radii of about 1 m for the corona. This implies α in the range 5.7° to 18° and I in the range 10^{11} A/s to a few times that.

III. CONCLUSIONS

This note has presented a simple model which demonstrates the plausibility of a maximum rate of rise of leader and return stroke currents of the order of 10^{11} A/s. The geometric model is crude, compared to what one might expect from detailed calculations of the surface at which the electric field approaches the uniform field breakdown level. Clearly the rate of rise calculations that were done for the conical geometry in this note should be continued using more complex geometries. One of those geometries should be the shape that was calculated in Reference 5 as the breakdown field boundary for a stationary, thin breakdown channel. Other simple boundaries should be considered, with the caveat that a more complex charge distribution is needed to avoid singularities in dI/dt.

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