Dielectric Strength Notes
Note 16

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D.C. Breakdown Voltages of Non-Uniform Gaps in Air

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One of the minor irritations of my life has been the fact that while approximate calculations of the breakdown volts of practical sphere/sphere and cylinder/cylinder gaps gives reasonable agreement to 10-20 per cent, when more accurate attempts are made to compare experiment with crude theory, reality seems strangely perverse. In addition to this, for pressurised gaps the plot of breakdown voltage against pressure is significantly non-linear. This non-linearity is distinct from the one that develops about 10 to 15 atmospheres, which, amonst other things, depends on the nature of the electrodes.

As a result of a succession of pressurised gaps which invariably just failed to meet their designed operating range, I was driven to derive a relatively simple picture of what was going on and as a result of this performed a series of measurements covering the range of parameters of practical interest to myself. When analysed these appeared to support the relatively simple picture proposed with satisfying accuracy. However, in analysing the effects of a fairly small correction, it became apparent that reality must be significantly more complex than the theory allowed it to be. However, in the last analysis a simple empirical approach has been derived which enables the breakdown voltage of a large range of pressurised air gaps to be calculated to some 2 or 3 per cent.

The starting point is the relationship for the DC breakdown field of a uniform field gap which can take the form

$$E = 24.5\, p \;+\; 6.7\, p^{1/2} / d^{1/2} \quad \text{kV/cm}$$

where p is the pressure in atmospheres and d is the spacing in centimetres. An approximate derivation of the form of this relation can be obtained by considering a Townsend avalanche which after 20 or so e foldings creates sufficient charge to start seriously modifying the field imposed by the electrodes. This then leads to the start of a modified discharge pattern such as the formation of a streamer. If the distance (obtained from the net ionisation coefficient $(\alpha - \beta)\, p$) is less than the electrode spacing, then a breakdown occurs. If, however, the electrodes are closer together than this distance, a breakdown does not take place. It should be stressed that using published data, the numerical agreement is not by any means perfect but it is certainly in the correct street. On this picture breakdown in diverging fields is modified because the avalanche is moving out into an ever-lower field and if the twentieth generation is not reached before the field has fallen too low, a breakdown will not occur. Consideration of the net ionisation curves suggested that if the field falls to about 80 per cent of the field on the electrode within the twentieth generation distance the avalanche peters out. For non-uniform gaps this picture suggests that the breakdown field at the surface should be taken to be

$$E = 24.5\ p + 6.7\ p^{1/2}\ \beta\ /\ r_{eff}^{\ 1/2} \quad kV/cm$$

where

$$r_{eff} = 0.115\ r \quad \text{spheres}$$

$$r_{eff} = 0.23\ r \quad \text{cylinders} .$$

The coefficients were determined from experimental data with spheres and cylinders separated by rather more than their radius and agrees well with the expectations of the theory outlined above. The coefficient β was included to allow for the fact that as the spheres or cylinders approached each other the field between becomes more uniform and eventually does not fall below 80 per cent. Thus the factor β tends to 1 as the spacing (d cms) becomes large compared to the radius of the electrodes and one of the aims of the experiments was to evaluate it for small gaps and to check it against theory. In order to complete the calculation of the breakdown voltage of a gap, the published expressions for the field enhancement on the axis of the electrodes is used. For the maximum field on the electrodes, the factor f is given in Figures 1 and 2 for spheres and cylinders respectively, where f is defined as

$$f = E_{max} / (V/d) .$$

As an example of the mode of use of the expression, the case of a sphere/sphere gap is considered where the radius of the spheres is 1 cm and the separation is 2 cm. The expression for the breakdown field is

$$E = 24.5\ p + 19.8\ p^{1/2} \quad kV/cm,$$

the factor β being 1 at this separation. For 1 atmosphere air $E = 44.3$ kV/cm and using Fig. 1 $f = 1.77$, giving an effective uniform gap separation of 1.13 cm and a breakdown voltage of 50 kV. As can be seen the breakdown voltage is significantly non-linear with pressure: in the example treated above the breakdown voltage at 4 atmospheres is calculated to be 155 kV.

A fairly extensive experimental programme was undertaken to check the proposed treatment and to obtain the value of β

for small gaps. Three radii of spheres were used, namely 0.635, 1.14, and 1.91 cm. and spacings of 0.5, 1.0, 1.5, 2.0 and 2.5 cm. The sphere/sphere gap could be pressurized and data was obtained at 1, 2 and 3 atmospheres absolute, subject to a voltage limitation, and no data were taken for breakdown voltages over 110 kV. For the experiments with cylinder/cylinder gaps (the actual experimental set up was cylinder/plane) three radii were again used, namely 0.64, 0.1285 and 2.035 cm., with separations of 1, 2, 3 and 4 cm. However, these gaps could not be pressurised, so all the measurements were at atmospheric pressure. However, a small series of experiments was performed with one rod/rod gap which could be pressurised and these results agreed well with the theory.

The voltage measurements were made with a calibrated voltage divider and had an error of the order of 1 per cent.

Including errors in the density of the air in the gap, the distance between the electrodes, and errors in calculating the large numbers of measurements, the error in the experimental data is estimated to be $\pm$ 1.8 per cent.

Using the β correction curve given in Fig. 3, the mean difference between the theory and the experimental results is given in Table I, as well as the standard deviations between the two. One comment which should be made with regard to the cylindrical data is that a polarity effect was observed. The measurements were made with a cylinder over a plane and it was found that the positive breakdown voltages were some 3 per cent higher than those for the negative voltage. Both the theory and the data quoted are for the negative polarity which of course would be that applying to a cylinder/cylinder gap.

TABLE I

	Radius (cms)	Difference Measurement to Theory (%)	Standard Deviation (%)
Spheres	0.635	+ 0.3	2.5
	1.14	+ 1.5	1.8
	1.91	+ 0.5	2.3
Cylinders	0.64	+ 1.0	2.1
	1.285	- 0.9	2.0
	2.035	+ 1.1	1.0

As is shown by Table I, the weighted mean difference between measurement and theory is only some 0.6 per cent and the standard deviation not much greater than the estimated measurement errors.

With regard to the correction factor β given in Fig. 3, the dotted curve shown is for a calculation for spheres at 1 atmospheric pressure, and once again agreement is very good considering that errors in experimental values correspond to something like $\pm$ 6 per cent in β on this plot. However, the figure 3 conceals two fairly serious failures. The first of these is that for the data for spheres, a pressure dependency of β would be expected but the data shows no such effect and hence the points plotted are the average of the three pressures used in the experiments. The second failure is that the detailed application of the theory would suggest that the correction curve would be shifted to the right by a factor of two for cylinders compared with the curve for spheres. Once again there is no real evidence for such a shift apart from the one point for $r = 0.64$. The other points for this radius are off to the right of the graph and average out to $\beta = 1$ as would be expected. Unfortunately the errors are greatest for this point and probably no significance can be attached to it being low.

Thus after a very satisfactory degree of agreement between experiment and theory, the details of the correction for only mildly non-uniform gaps fails to display detailed agreement. While there are arguments which can account for some of the differences, it is probably simpler to regard the β correction as more or less purely experimental. Again it is probably worth stating that substantial errors in β relfect only weakly in the final calculated breakdown voltage.

In conclusion I would like to express my appreciation of Mr. Brian Harle's help in obtaining the experimental data. Not only did he assist in a rather lengthy and boring series of measurements, but he also served to keep my scientific standards from wilting under the tedium.

$f\left(\frac{d}{R}\right)$ FOR EQUAL SPHERES

$f \equiv E_{Max} / V/d$

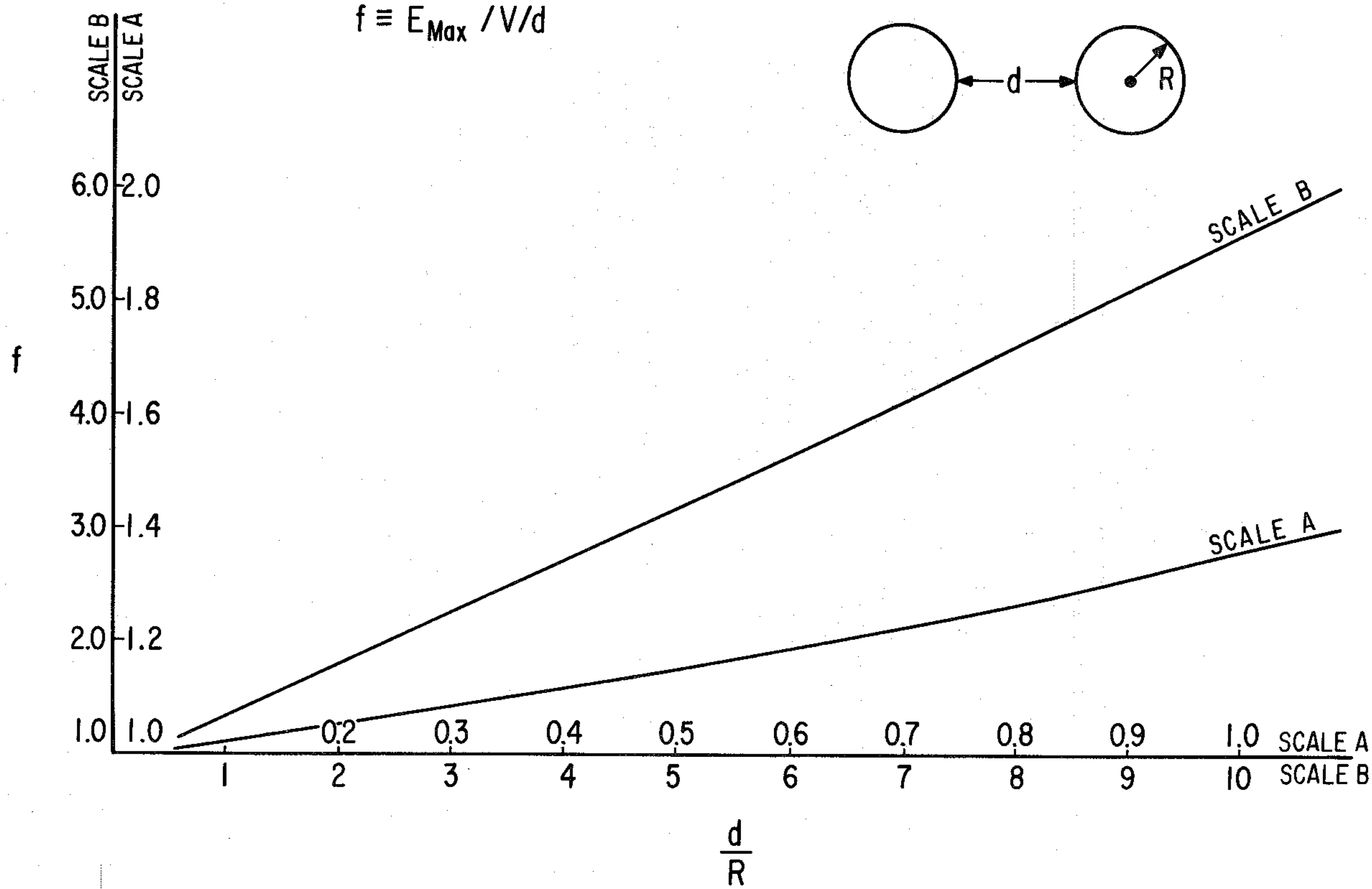
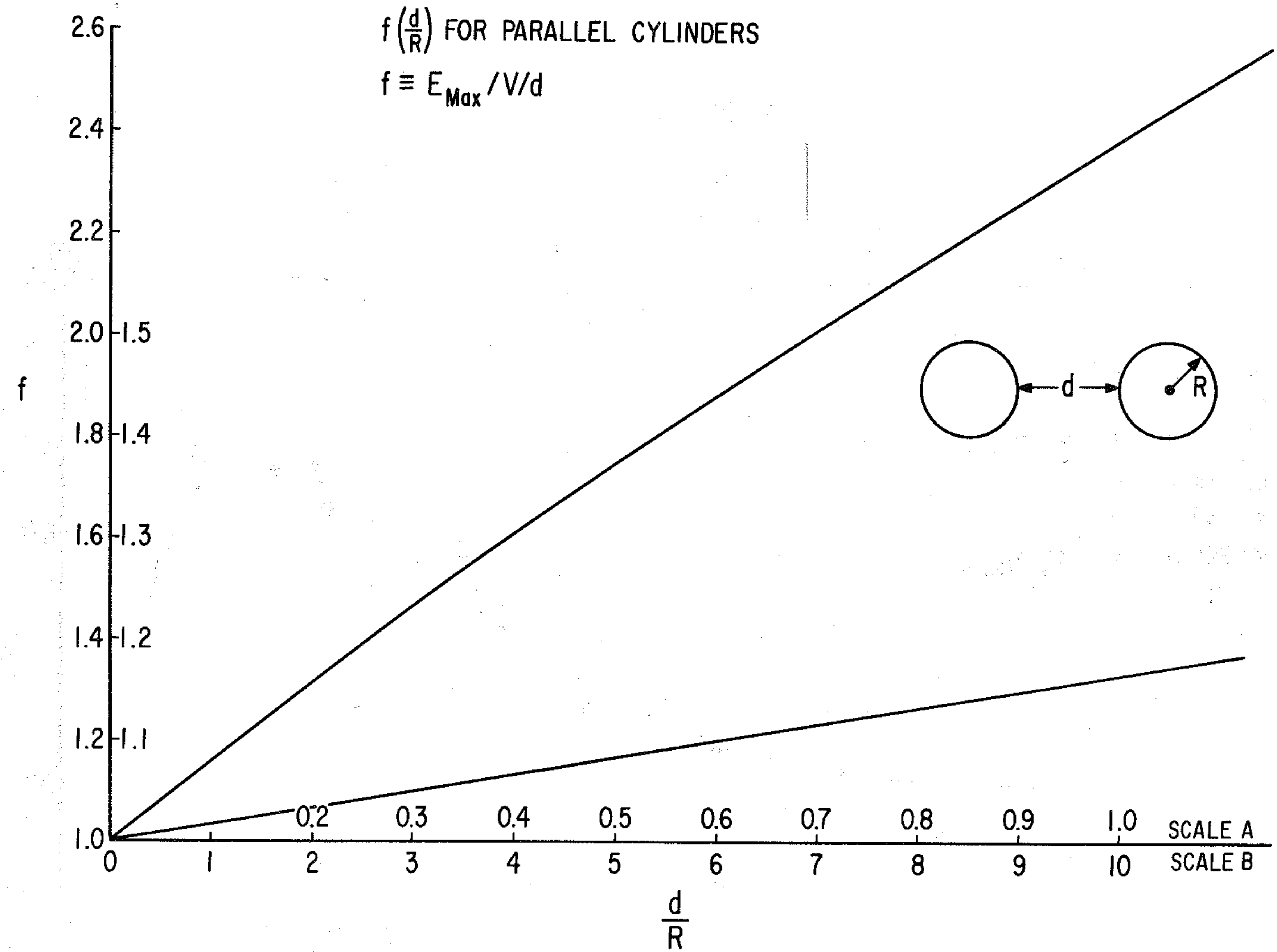


FIGURE 1



f (d/R) FOR PARALLEL CYLINDERS
f ≡ E_Max / V/d
f
2.6
2.4
2.2
2.0 1.5
1.8 1.4
1.6 1.3
1.4 1.2
1.2 1.1
1.0
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 SCALE A
0 1 2 3 4 5 6 7 8 9 10 SCALE B
d
R
d/R

FIGURE 2

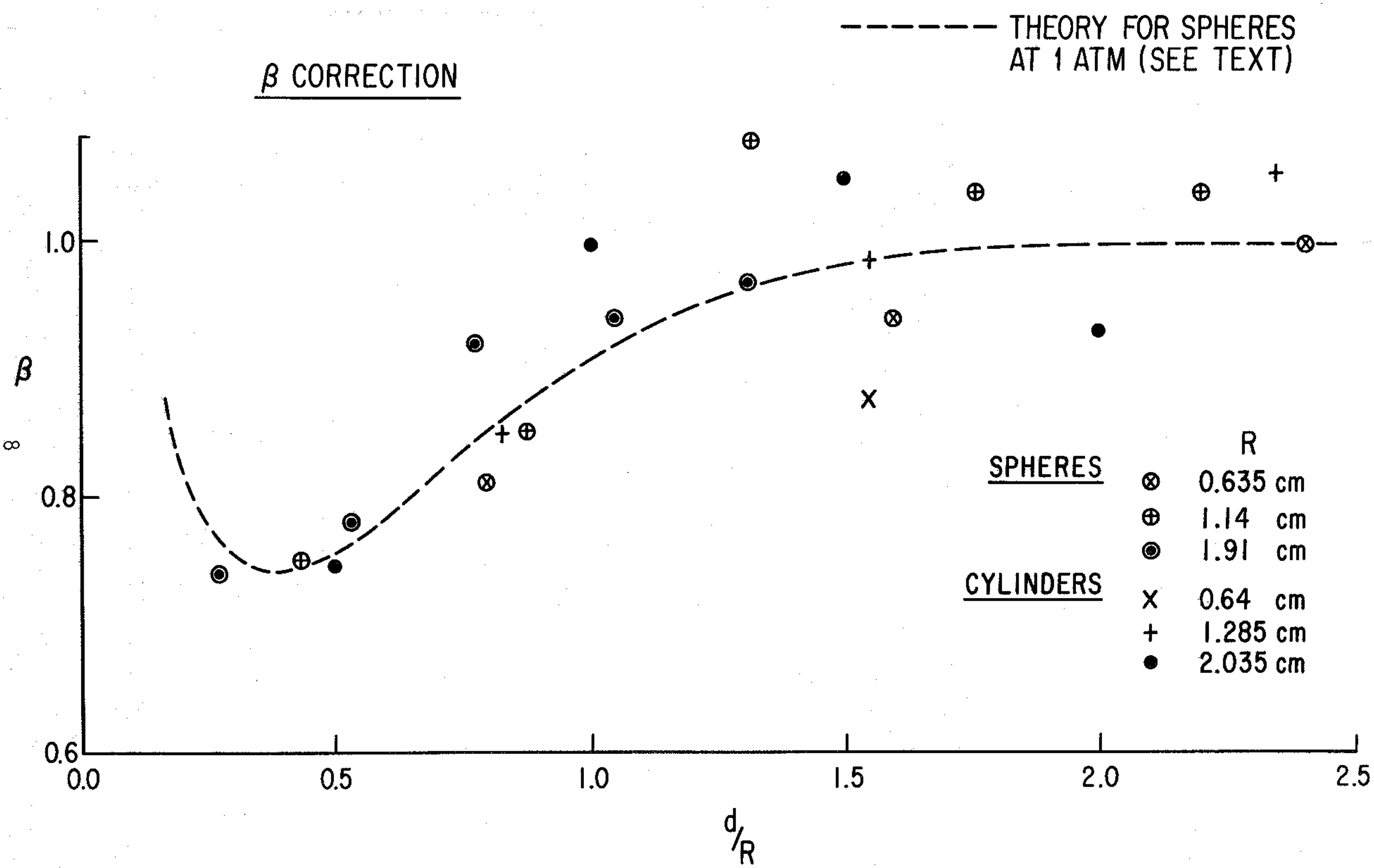

β CORRECTION
THEORY FOR SPHERES
AT 1 ATM (SEE TEXT)
β
1.0
0.8
0.6
0.0
0.5
1.0
1.5
2.0
2.5
d/R
R
SPHERES
0.635 cm
1.14 cm
1.91 cm
CYLINDERS
0.64 cm
1.285 cm
2.035 cm

FIGURE 3