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Tradeoff Between Pulse Amplitude and Pulse Length From a Microwave-Pulse-Compression Source

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Abstract

Microwave pulse compression offers a way to illuminate targets with high-power-electromagnetic sources. This paper considers the tradeoff between short, high-amplitude pulses, and long, low-amplitude pulses for optimal target response.

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1. Introduction

In designing microwave weapons one needs to consider such questions as frequency, pulse amplitude, power, pulse length, and energy. These need to be traded off against each other for optimum effect (disruption or damage) on a target.

An interesting class of sources is based on microwave pulse compression. Some lower power source is used to feed power into a resonant cavity. As discussed in [5], there is a tradeoff between microwave power gain

$$G_s = \frac{P}{P_0} \equiv \frac{\text{power in cavity (after ringup)}}{\text{power in microwave source}} \tag{1.1}$$

and the length of a waveguide cavity as

$$G_s \equiv \frac{1}{4\alpha\ell}$$

$$\alpha \equiv \text{attenuation constant (units } m^{-1}\text{) in } e^{\pm\alpha z} \text{ in waveguide} \tag{1.2}$$

$$\ell \equiv \text{waveguide length}$$

This does not include losses in the waveguide ends, switch(es), irises [6], and other connections. Here the power in the waveguide cavity by considering the waveguide resonance as comprised of two oppositely propagating waves. The power is approximately the same in the two waves (low-loss approximation).

For a given frequency f_s there is a certain guide wavelength λ_g, and the waveguide is some number N_g of half-guide-wavelengths long. The amount of stored energy is [6]

$$W = \frac{2LP}{\text{v}_{gr}} \quad , \quad \ell = \frac{N_g \lambda_g}{2}$$

$$\frac{\text{v}_{ph}}{\lambda_g} = \frac{c}{\lambda}$$

$$\lambda \equiv \text{free space wavelength (or in dielectric medium)}$$

$$c \equiv \text{free space velocity (or indielectric medium)}$$

$$\text{v}_{ph} \equiv \text{phase velocity} \tag{1.3}$$

$$\text{v}_{gr} \equiv \text{group velocity (for narrow band signal)}$$

Then using [10]

$$\mathrm{v}_{gr} = \frac{c^2}{\mathrm{v}_{ph}} = c\frac{\lambda}{\lambda_g} \tag{1.4}$$

we have

$$W = \frac{2\ell}{c}\frac{\lambda_g}{\lambda} P = \frac{1}{2\alpha c}\frac{\lambda_g}{\lambda} P_0 \tag{1.5}$$

where λ_g / λ is greater than one, but not too much greater for practical cases. Also, α becomes large near cutoff [10].

So (1.5) tells us that the energy, W, in the cavity is approximately independent of the cavity length, ℓ, while the power available is inversely proportional to the cavity length. (Note that cavity-end effects are not included here.) For a given W we can have *short, big* pulses or *small, long* pulses, but *not* big, long pulses.

For a given N_g, if one extracts all the energy, W, in one pass (at one end), then the number of guide wavelengths in the pulse is N_g. However, the temporal length of the extracted pulse is somewhat larger due to the group velocity, v_{gr}, which is less than v_{ph} in the waveguide. This gives a temporal width of the extracted pulse, T, (with) approximately a rectangular envelope [8]) of approximately

$$T \simeq 2\frac{\ell}{\mathrm{v}_{gr}} = \frac{2\ell}{c}\frac{\mathrm{v}_{ph}}{c} = \frac{2\ell}{c}\frac{\lambda_g}{\lambda} \tag{1.6}$$

This increases the number of cycles in the output pulse from N_g to

$$N_{out} = Tf_s = T\frac{c}{\lambda} = \frac{2\ell}{\lambda}\frac{\lambda_g}{\lambda} = N_g\left[\frac{\lambda_g}{\lambda}\right]^2 \tag{1.7}$$

So the pulse is stretched by an amount $[\lambda_g / \lambda]^2$. The power out is reduced accordingly.

Note that if the power is extracted half-way along the waveguide, so that the number of wavelengths is halved, the power out is also doubled [7]. This is consistent with the energy relationship in (1.5).

2. General Response of Target

As discussed in [1] systems of interest (missiles, aircraft, computers, etc.) are characterized by resonant response, making it quite advantageous to tune the microwave radiator to such a resonance, thereby maximizing the response. Previous papers [2-4] have considered the response enhancement by appropriately matching the excitation waveform to a target resonance.

To better understand the response of a canonical target, let us consider the response to a single resonance in a transfer function as

$$
\begin{aligned}
&\tilde{T}(s) = T_0\left[s-s_0\right]^{-1} + T_0^*\left[s-s_0^*\right]^{-1} \\
&\sim \;\equiv\; \text{Laplace transform (two sided) over time, } t \\
&s \equiv \Omega + j\omega \equiv \text{Laplace-transform variable or complex frequency} \\
&* \equiv \text{complex conjugate} \\
&s_0 \equiv \Omega_0 + j\omega_0 \equiv \text{complex resonant frequency} \\
&\Omega_0 < 0 \text{ (for causal response)}
\end{aligned}
\tag{2.1}
$$

The response of such a system to a delta function, $\delta(t)$, is

$$
T(t) = \left[T_0 e^{s_0 t} + T_0^* e^{s_0^* t}\right]u(t) = 2\left[\mathrm{Re}(T_0)\cos(\omega_0 t) - \mathrm{Im}(T_0)\sin(\omega_0 t)\right]e^{\Omega_0 t}u(t) \tag{2.2}
$$

For some given excitation, say an electric field $E(t)$, one can compute the response of the system as

$$
\begin{aligned}
&V(t) = E(t) \circ T(t) \\
&\tilde{V}(s) = \tilde{E}(s)\tilde{T}(s) \\
&\circ \equiv \text{convolution with respect to time}
\end{aligned}
\tag{2.3}
$$

This of course, applies to some particular polarization and direction of incidence of an incident wave. The natural frequencies (s_0), are only a function of the target, and invariant to the incident wave parameters. (See a plethora of papers and book chapters on the singularity expansion method (SEM).)

3. Properties of Rectangular Modulated Sinusoidal Pulse

Previous papers [2-4] have considered various waveforms for exciting maximum response of targets. The general conclusion is that much is to be gained by matching the excitation frequency to the target-response frequency (resonance). In [3] norms are used to compare the system response to various excitation waveforms.

For the present discussion let us assume that we are given a certain amount of energy (per unit area) available at the target as

$$W_t = \int_{-\infty}^{\infty} \frac{1}{Z_0} E^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{Z_0} |\tilde{E}(j\omega)|^2 \, d\omega \quad \text{(Parseval theorem)} \tag{3.1}$$

This is cast as a 2-norm

$$W_t = \frac{1}{Z_0}\left\| E(t) \right\|_2^2 = \frac{1}{2\pi Z_0}\left\| \tilde{E}(j\omega) \right\|_2^2 \tag{3.2}$$

So we can consider the 2-norm

$$\left\| E(t) \right\|_2 = \frac{1}{\sqrt{2\pi}} \left\| \tilde{E}(j\omega) \right\|_2 \tag{3.3}$$

as constrained by the source. We wish now to maximize the response under a constraint on the 2-norm.

From the discussion in Section 1 let us take the excitation as a rectangularly modulated sinusoidal pulse of the form

$$\begin{aligned} E(t) &= E_0 \cos(\omega_s t)\left[u(t) - u(t-T)\right] \\ &= \frac{E_0}{2}\left[e^{s_s t} + e^{-s_s t}\right]\left[u(t) - u(t-T)\right] \\ s_s &= j\omega_s \\ \omega_s &= 2\pi f_s \end{aligned} \tag{3.4}$$

One could add a phase shift to the term, but for high-Q targets this will make little difference. For convenience we can define

$$
\begin{aligned}
E(t) &\equiv E_1(t) + E_2(t) \\
E_1(t) &= \frac{E_0}{2}\left[e^{s_s t} + e^{-s_s t}\right]u(t) \\
E_2(t) &= -\frac{E_0}{2}\left[e^{s_s t} + e^{-s_s t}\right]u(t-T) \\
&= -\frac{E_0}{2}\left[s^{s_s[t-T]}u(t-T)e^{s_s T} + e^{-s_s[t-T]}u(t-T)e^{-s_s T}\right]
\end{aligned}
\tag{3.5}
$$

This gives in complex-frequency domain

$$
\begin{aligned}
E_1(s) &= \frac{E_0}{2}\left[\left[s-s_s\right]^{-1} + \left[s-s_s\right]^{-1}\right] \\
E_2(s) &= -\frac{E_0}{2}\left[\left[s-s_s\right]^{-1} e^{-[s-s_s]T} + \left[s-s_s\right]^{-1} e^{[s+s_s]T}\right] \\
E(s) &= \frac{E_0}{2}\left[\frac{1-e^{-[s-s_s]T}}{s-s_s} + \frac{1-e^{[s+s_s]T}}{s+s_s}\right]
\end{aligned}
\tag{3.6}
$$

Note that for s near s_s we have

$$
\tilde{E}(s) \simeq \frac{E_0}{w}\left[T + \frac{1-e^{2s_s T}}{2s_s}\right]
\tag{3.7}
$$

Noting that

$$
\left|s_s\right| = \left|\omega_s\right| = 2\pi f_s = 2\pi\frac{N_{out}}{T}
\tag{3.8}
$$

then for large N_{out} we have

$$
\tilde{E}(s) \simeq \frac{E_0 T}{2} \quad \text{for } s \to s_0
\tag{3.9}
$$

In norm sense the incident wave is characterized by

$$
\begin{aligned}
\|E(t)\|_\infty &= E_0 \\
\|E(t)\|_2 &= E_0\left[\int_0^T \cos^2(\omega_s t)\,dt\right]^{1/2} = \left[Z_0 W_t\right]^{1/2} \\
&\simeq E_0\left[\frac{T}{2}\right]^{1/2} = E_0\left[\frac{N_{out}}{2 f_s}\right]^{1/2} \\
\|E(t)\|_1 &= E_0 \int_0^T \left|\cos(\omega_s t)\right| dt \\
&\simeq E_0 \;\; 4\, N_{out} \int_0^{\frac{\pi}{2\omega_s}} \cos(\omega_s t)\,dt \\
&= E_0\, N_{out}\;\; 4\, \frac{\sin(\omega_s t)}{\omega_s}\Bigg|_0^{\frac{\pi}{2\omega_s}} = E_0\,\frac{4\, N_{out}}{\omega_s} \\
&= E_0\,\frac{2}{\pi}\,\frac{N_{out}}{f_s} = E_0\,\frac{2T}{\pi}
\end{aligned}
\tag{3.10}
$$

4. Peak Voltage Response of Target to Rectangular Modulated Sinusoidal Pulse

Now consider the response of the target to the (3.4) environment under the constraint of fixed incident energy, W_t , as in (3.1). This corresponds to fixed 2-norm as in (3.6).

The system response from (2.3) is

$$
\begin{aligned}
V(t) &= E(t) \circ T(t) \\
\tilde{V}(s) &= \tilde{E}(s)\,\tilde{T}(s)
\end{aligned}
\tag{4.1}
$$

where $V(t)$ is the voltage waveform at some circuit of interest. System failure at its lowest levels is transient upset for which the peak or ∞-norm is relevant. As one increases the voltage at the circuit (say by another order of magnitude) one can encounter a voltage breakdown of a semiconductor device. Again, the ∞-norm is important. If, on the other hand, the total energy deposited in a circuit element at low amplitude but long time is significant, then the 2-norm becomes significant. In this latter case one should limit the integration time to a thermal decay time of the circuit element.

Combining the environment with the system response (transfer function in (2.1)) gives

$$\tilde{V}(s) = \frac{E_0}{2}\left[\frac{1-e^{-[s-s_s]T}}{s-s_s} + \frac{1-e^{-[s+s_s]T}}{s+s_s}\right]\left[T_0\left[s-s_0\right]^{-1} + T_0^*\left[s-s_0^*\right]^{-1}\right] \tag{4.2}$$

The time-domain version of this is given by the inverse Laplace transform from standard tables [9] as

$$\begin{aligned} V(t) = \frac{E_0}{2}\Bigg[& T_0\left[\frac{e^{s_s t} - e^{s_0 t}}{s_s - s_0} - \frac{e^{-s_s t} - e^{s_0 t}}{s_s + s_0}\right]u(t) + T_0^*\left[\frac{e^{s_s t} - e^{s_0^* t}}{s_s - s_0^*} - \frac{e^{-s_s t} - e^{s_0^* t}}{s_s + s_0^*}\right]u(t) \\ & - T_0\left[\frac{e^{s_s[t-T]} - e^{s_0[t-T]}}{s_s - s_0}\, e^{s_s T} - \frac{e^{-s_s[t-T]} - e^{s_0[t-T]}}{s_s + s_0}\, e^{-s_s t} - \right]u(t-T) \\ & - T_0^*\left[\frac{e^{s_s[t-T]} - e^{s_0^*[t-T]}}{s_s - s_0^*}\, e^{s_s T} - \frac{e^{-s_s[t-T]} - e^{s_0^*[t-T]}}{s_s + s_0^*}\, e^{-s_s t} - \right]u(t-T)\Bigg] \end{aligned} \tag{4.3}$$

Now make some approximations, including

$$\begin{aligned} & |\Omega_0| << \omega_0 && \text{(highly resonant target)} \\ & s_s = j\omega_s = j\omega_0 && \text{(tune to resonance)} \\ & s_s - s_0 = -\Omega_0 && \text{(positive)} \\ & s_s + s_0^* = \Omega_0 && \text{(negative)} \\ & s_s + s_0 = 2j\omega_s + \Omega_0 \simeq 2j\omega_s && \\ & s_s - s_0^* = 2j\omega_s - \Omega_0 \simeq 2j\omega_s && \end{aligned} \tag{4.4}$$

Then (4.3) simplifies to

$$V(t) \simeq \frac{E_0}{2}\left[T_0\, e^{j\omega_s t} + T_0^* e^{-j\omega_s t}\right]\left[\frac{1-e^{\Omega_0 t}}{|\Omega_0|}u(t) - \frac{1-e^{\Omega_0[t-T]}}{|\Omega_0|}u(t-T)\right] \tag{4.5}$$

Looking at these terms let T encompass many cycles. Then we can approximate the envelope from

$$\sup_t\left[T_0\, e^{j\omega_s t} + T_0^* e^{-j\omega_s t}\right] = 2|T_0| \tag{4.6}$$

so that the envelope of $V(t)$ is just

$$\text{envelope}\left(V(t)\right) \simeq \frac{E_0|T_0|}{|\Omega_0|}\left[\left[1-e^{\Omega_0 t}\right]u(t) - \left[1-e^{\Omega_0[t-T]}\right]u(t-T)\right] \tag{4.7}$$

The peak voltage is then given by the peak of this function. This occurs at $t = T$, giving

$$\|V(t)\|_\infty \simeq \frac{E_0|T_0|}{|\Omega_0|}\left[1-e^{\Omega_0 T}\right] \tag{4.8}$$

For $t > T$ the second term in (4.7) reduces the voltage envelope. For short excitation pulses we have

$$\begin{aligned} &|\Omega_0 T| << 1 \\ &\|V(t)\|_\infty \simeq E_0 T|T_0| \end{aligned} \tag{4.9}$$

and the target is not fully rung up. For long excitation pulses we have

$$\begin{aligned} &|\Omega_0 T| >> 1 \\ &\|V(t)\|_\infty \simeq E_0\frac{|T_0|}{|\Omega_0|} \end{aligned} \tag{4.10}$$

Returning to the 2-norm (energy) constraint on the source in (3.6) we have

$$\frac{\|V(t)\|_\infty}{\|E(t)\|_2} \simeq T_0\,\frac{1-e^{\Omega_0 T}}{|\Omega_0|}\left[\frac{2}{T}\right]^{1/2} \tag{4.11}$$

For short excitation pulses we have

$$\begin{aligned} &|\Omega_0 T| << 1 \\ &\frac{\|V(t)\|_\infty}{\|E(t)\|_2} \simeq |T_0|[2T]^{1/2} \end{aligned} \tag{4.12}$$

suggesting that longer pulses are better. For long excitation pulses we have

$$
\begin{aligned}
&\left|\Omega_0 T\right| \ll 1 \\
&\frac{\|V(t)\|_\infty}{\|E(t)\|_2} \simeq \frac{|T_0|}{|\Omega_0|}\left[\frac{2}{T}\right]^{1/2}
\end{aligned}
\tag{4.13}
$$

suggesting that shorter pulses are better.

To optimize the pulse length, note that Ω_0 is fixed and rearrange (4.11) in the form

$$
\begin{aligned}
\frac{\|V(t)\|_\infty}{\|E(t)\|_2} &= |T_0|\,\frac{1-e^{-a}}{a^{1/2}}\left[\frac{2}{\Omega_0}\right]^{1/2} \\
a &= \left|\Omega_0\, T\right|
\end{aligned}
\tag{4.14}
$$

By differentiating with respect to *a* we find zero derivative at

$$
\begin{aligned}
2a &= e^{a}-1 \ , \ a \neq 0 \\
a &\simeq 2.52 \\
\frac{1-e^{-a}}{a^{1/2}} &\simeq 0.64
\end{aligned}
\tag{4.15}
$$

so the optimum pulse width is

$$
T \simeq 2.52\left|\Omega_0\right|^{-1}
\tag{4.16}
$$

with

$$
\frac{\|V(t)\|_\infty}{\|E(t)\|_2} \simeq 0.9\left|T_0\right|\left|\Omega_0\right|^{-1/2}
\tag{4.17}
$$

5. Peak Energy Response of Target to Rectangular Modulated Sinusoidal Pulse

For completeness, let us consider the energy response which takes the form

$$\mathrm{U} = \frac{\|V(t)\|_{2,t}}{\|E(t)\|_{2,t}} = \frac{\|\tilde{V}(j\omega)\|_{2,\omega}}{\|\tilde{E}(j\omega)\|_{2,\omega}} \tag{5.1}$$

where the 2-norms can be evaluated over time or frequency (from the Parseval theorem) as convenient. From (4.1) we can also write the ratio as

$$\mathrm{U} = \frac{\|E(t) \circ T(t)\|_{2,t}}{\|E(t)\|_{2,t}} = \frac{\|\tilde{E}(j\omega)\tilde{T}(j\omega)\|_{2,\omega}}{\|\tilde{E}(j\omega)\|_{2,\omega}} \tag{5.2}$$

To make a long story short, note that it takes a certain time, proportional to $|\Omega_0|^{-1}$, to ring up the target. For times long compared to this a steady state is reached. In this case, we can approximate the terms with $T > t$ and ω near ω_s as

$$\begin{aligned} \left|\tilde{V}(j\omega)\right| &\simeq \frac{E_0}{2} T_0 \left|\Omega_0\right|^{-1} \\ \left|\tilde{E}(j\omega_0)\right| &\simeq \frac{E_0}{2} \end{aligned} \tag{5.3}$$

There is another equal pair for $-\omega_0$. For long pulses the energy is just power times time (which divides out in the ratio), giving

$$\mathrm{U} \simeq T_0 \left|\Omega_0\right|^{-1} \tag{5.4}$$

The total energy deposited is proportional to $\mathrm{U}^2 \mathrm{E}_0^2 T$, where T should be limited to thermal decay times. Note that the source has $\mathrm{E}_0^2 T$ limited, so U^2 is proportional to the total energy delivered to the circuit (assumed linear).

6. Concluding Remarks

The foregoing analysis shows some limitations for microwave pulse-compression sources (with antennas) in their potential effects on targets. Basically, the total energy incident on the target is limited. One can have large, short pulses (for upset and damage), or small, long pulses (for maximum energy delivery to linear loads).

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