Circuit and Electromagnetic System Design Notes

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Switched Oscillators Combined With Transformers to Give Design Flexibility

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Abstract

This paper considers the use of transformers to control the matching of a switched oscillator to an antenna. This gives flexibility in design trades between amplitude and pulse width. It also allows variation in the oscillator design for increased energy.

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1. Introduction

The switched oscillator has proved to be a useful source for mesoband radiation. By maintaining a large antenna-to-oscillator impedance ratio, an approximate damped sinusoidal waveform is produced. This leaves the question of how large a Q, or the number N of cycles to e^{-1}, is desired for a particular application. This leaves some flexibility in the design of a switched oscillator.

As indicated in Fig. 1.1 we have a quarter-wavelength transmission line of characteristic impedance Z_c, charged to a potential $-V_0$. This is switched to ground at one end with the load at the other end. Now insert a transformer between the oscillator and the antenna load. Let this transformer be described by a transfer function T where

$$T = \frac{V_a}{V_{in}} = \frac{I_{in}}{I_a} \tag{1.1}$$

The switched oscillator now drives an impedance

$$Z_{in} = T^{-2} Z_a \tag{1.2}$$

An ideal transformer could be considered to have T as the ratio of secondary to primary turns.

The switched oscillator has stored energy

$$U_0 = \frac{1}{2} \mathrm{C}\, V_0^2 = \frac{1}{2} \frac{t_r}{Z_c} V_0^2 \quad , \quad t_r = \frac{1}{4 f_0} \equiv \text{transit time} \quad , \quad f_0 \equiv \text{resonant frequency} \tag{1.3}$$

This energy is a limiting factor in how much can be sent to the antenna. As discussed in [1], this type of switched oscillator has a resonant amplitude of $(4/\pi)\ V_0$ for the primary resonant frequency. The available energy is

$$U_{os} = \frac{8}{\pi^2}\, U_0 \simeq 0.8\, U_0 \tag{1.4}$$

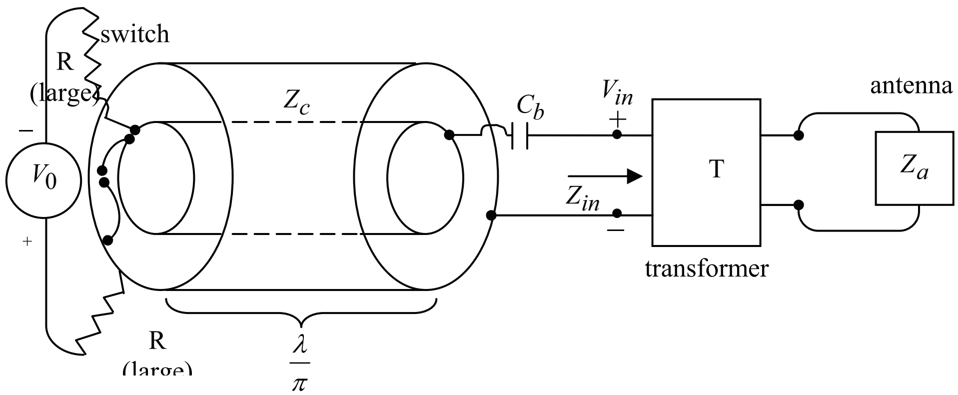


Fig. 1.1 Switched Oscillator Driving Transformed Antenna Load

There is also the energy stored in the blocking capacitor C_b, which is not included here.

For a waveform into our antenna its energy is proportional to amplitude squared times pulse width. Before introducing the transformer we have

$$U_{os} = u_1 V_a^2 N = \frac{u_1}{\pi} V_a^2 Q \quad , \; u_1 \geq 0 \tag{1.5}$$

Conserving energy we have

$$U_{os} = u_1 \left[T \, V_a \right]^2 \mathrm{NT}^{-2} \tag{1.6}$$

So, as we increase the voltage to the antenna we decrease the pulse with (N and Q) by T^{-2}. This gives as some feel for the design trades available to us.

2. Effect of Introducing Transformer

Without the transformer, typically $Z_a >> Z_c$ so that V_{in} is close to $2V_0$. With a reflection coefficient [3]

$$\rho = \frac{1 - \dfrac{Z_c}{Z_a}}{1 + \dfrac{Z_c}{Z_a}} \tag{2.1}$$

a damped sinusoid decays to e^{-1} in

$$N = \frac{Q}{\pi} = -\frac{1}{2\ell n(\rho)} \tag{2.2}$$

The voltage incident on the antenna is then

$$V_a = [1+\rho]V_0 \tag{2.3}$$

For large N we have

$$\rho \simeq 1 - \frac{1}{2}\frac{Z_c}{Z_a} \quad , \quad N = -\frac{1}{2\ell n(\rho)} = \frac{Z_a}{Z_c} \quad , \quad V_a \simeq 2V_0 \tag{2.4}$$

as we should expect.

From an energy point of view, the energy delivered to the antenna is

$$\begin{aligned} U_a &= \frac{u_3 V_a^2}{2 Z_a} N = \frac{u_3}{\pi}\frac{V_a^2}{2Z_a} Q \\ u_3 &> 0 \end{aligned} \tag{2.5}$$

From (2.4) we then have for large N

$$U_a \simeq \frac{u_3 V_a^2}{2 Z_c} \simeq u_3 \, 2\frac{V_0^2}{Z_c} \tag{2.6}$$

This can be compared to (1.4) and (1.5), showing the consistency between the energy delivered to the antenna, and that stored in the oscillator.

Now, introducing the transformer, we have a reflection coefficient at the transformer input as

$$\rho_t = \frac{1 - \dfrac{Z_c}{Z_{in}}}{1 + \dfrac{Z_c}{Z_{in}}} = \frac{1 - T^2 \dfrac{Z_c}{Z_{in}}}{1 + T^2 \dfrac{Z_c}{Z_{in}}} \tag{2.7}$$

For large N this becomes

$$N_t = \frac{Q_t}{\pi} = -\frac{1}{2\ell n(\rho)} \simeq \frac{Z_{in}}{Z_c} = T^{-2}\frac{Z_a}{Z_c} \tag{2.8}$$

So N is decreased by T^2. The voltage (peak) on the antenna is now

$$V_{at} = [1+\rho]V_0 T = \frac{2\,V_0\,T}{1+\frac{Z_c}{Z_{in}}} = \frac{2\,V_0\,T}{1+T^2\,\frac{Z_c}{Z_a}} \tag{2.9}$$

which for large N is

$$V_{at} \simeq 2\,V_0\,T \tag{2.10}$$

Then for comparison to (2.6) we have

$$\begin{aligned} U_{at} &= \frac{u_3\,V_{at}^2}{2\,Z_a} N_t = u_3 \frac{[2\,V_0\,T]^2}{Z_a} T^{-2} N \\ &= U_a \end{aligned} \tag{2.11}$$

So there is a fairly simple way to view the effect of the transformer via

$$V_a \to V_{at} \simeq N\,V_a \quad , \quad N_t \to N\,T^{-2} \tag{2.12}$$

For sufficiently large N this provides a simple scaling. For small N one can consider a more detailed result, still based on conservation of energy.

3. Types of Transformers

There are various ways to construct a transformer with voltage multiplication T. Our concern here is for frequencies in the general range of a GHz.

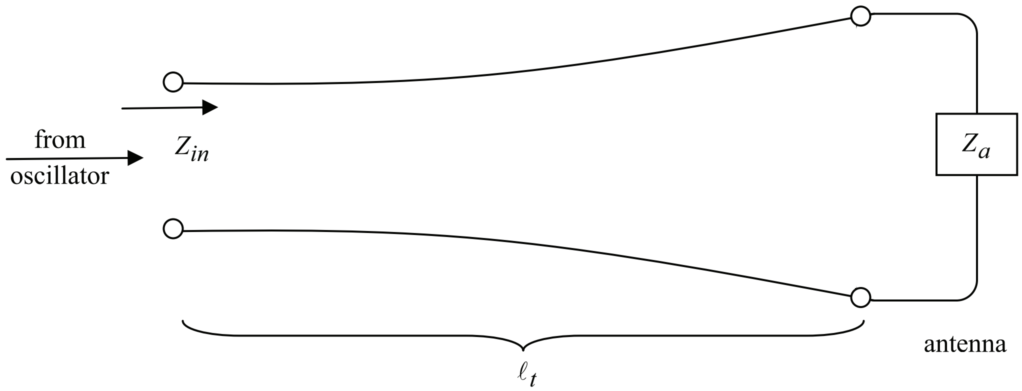
3A. Exponential-transmission-line transformer

An exponential variation of the transmission-line characteristic impedance makes a good transformer with an analytic closed-form expression for the transfer function (including low frequencies) [2]. It is quite appropriate for broadband pulses.

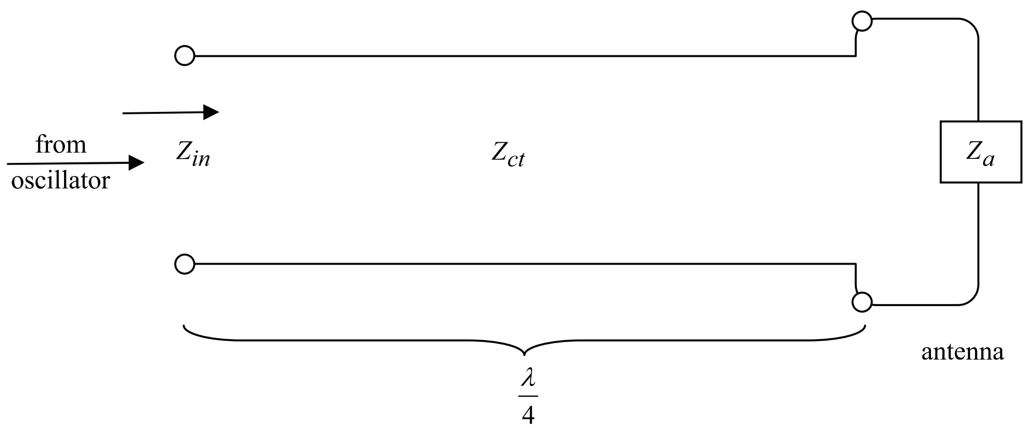
As in Fig. 3.1A the characteristic impedance begins at Z_{in} and grows exponentially with distance, ending up at Z_a. The length ℓ_t is chosen sufficiently large so that at the frequency f_0 there is negligible reflection.

3B. Quarter-wave transformer

A quarter-wave transformer is a very simple device it is a $\lambda / 4$ length of uniform transmission line with characteristic impedance



A. Exponential transformer



B. Quarter-wave transformer

Fig. 3.1 Transformer Examples

$$Z_{ct} = [Z_{in} Z_a]^{1/2} \tag{3.1}$$

with an input impedance Z_{in}. This device has a small bandwidth centered on f_0. As such it is appropriate for large N.

One aspect of the design is the fact that the input impedance to the transformer, Z_{in}, does not need to exist as the characteristic impedance of a transmission line. One can directly connect from the oscillator characteristic impedance Z_c to the transmission line of characteristic impedance Z_{ct}. (See also [4].)

3C. Other kinds of transformers

The essential characteristic of a transformer is conservation of power from input to output, going from one impedance to another. One can even have an old-style transformer where T represents a turns ratio (secondary to primary), whether with an aircore or permeable core. Such a transformer might save space in the case of a low f_0. An example might be an autotransformer (some windings common to both primary and secondary).

4. Varying Z_c to Increase Energy

Now that we understand how to match an oscillator to an antenna to obtain various amplitudes (V_a) and pulse widths (N), let us go back to the oscillator itself. The transformer shows us how to convert the oscillator stored energy into amplitudes and pulse widths for a given stored energy.

What then about the stored energy in the oscillator? How can we maximize this? For some desired f_0 we have an oscillator length $\lambda / 4$. So the (electrical) length is constrained. So what about the cross section? We know that the radius must be somewhat less than $\lambda / 4$ so that it must behave as a one-dimensional transmission line. What then shall we do within these constraints?

A recent paper gives one set of approaches to better filling the cross section and thereby reducing the oscillator characteristic impedance and increasing the stored energy [5]. The transformers discussed in the present paper can also be combined with this technique of lowering the oscillator characteristic impedance.

Another approach to filling the oscillator volume raises Z_c as in Fig. 4.1. The character is impedance is

$$Z_c = Z_w f_g \ , \quad Z_w = \left[\frac{\mu_0}{\varepsilon}\right]^{1/2} \ , \quad f_g = \frac{1}{2\pi} \ell n\left(\frac{\Psi_2}{\Psi_1}\right) \tag{4.1}$$

For a given maximum electric field of E_0 at the center conductor, the electric-field distribution is

$$E(\Psi) = E_0 \frac{\Psi_1}{\Psi} \tag{4.2}$$

The potential distribution is

$$\begin{aligned} V(\Psi) &= \int_{\Psi}^{\Psi_2} E(\Psi') d\Psi' = E_0 \Psi_1 \ell n\left(\frac{\Psi_2}{\Psi}\right) \\ V_0 &= V(\Psi_1) = E_0 \Psi_1 \ell n\left(\frac{\Psi_2}{\Psi_1}\right) = 2\pi f_g \Psi_1 E_0 \end{aligned} \tag{4.3}$$

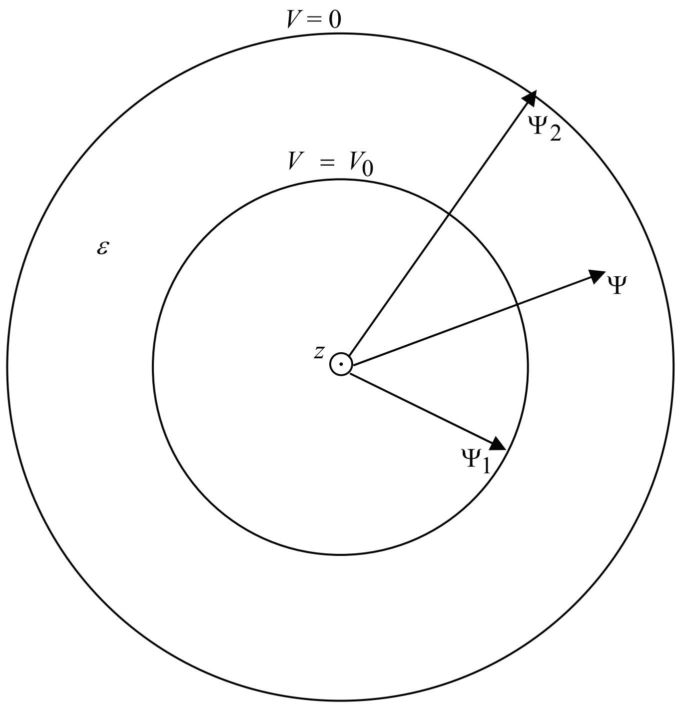


Fig. 4.1 High-Characteristic-Impedance Oscillator.

The stored energy is

$$U_0 = \frac{1}{2}\mathrm{C}\,V_0^2 = \frac{1}{2}\frac{t_r}{Z_c}V_0^2 = \frac{V_0^2}{8Z_c f_0} = \frac{\pi^2}{2}f_g^2\frac{\Psi_1^2E_0^2}{Z_c f_0} = \frac{\pi^2}{2}\frac{f_g}{Z_w}\frac{\Psi_1^2E_0^2}{Z_c f_0} \tag{4.4}$$

Here we see that the energy is increased for a given f_0 and ε by larger f_g (larger Z_c), radius Ψ_1, and electric field E_0 (dielectric breakdown). However, Ψ_2 is limited to be significantly less than the length. So we need to consider

$$X = f_g\Psi_1^2 = \frac{1}{2\pi}\ell n\left(\frac{\Psi_2}{\Psi_1}\right)\Psi_1^2 \tag{4.5}$$

This is maximized by setting

$$\frac{dX}{d\Psi_1} = 0 = \frac{1}{2\pi}\left[2\Psi_1\ell n\left(\frac{\Psi_2}{\Psi_1}\right) - \Psi_1\right] \quad , \quad \frac{\Psi_2}{\Psi_1} = e^{1/2} \simeq 1.65 \tag{4.6}$$

The stored energy (maximum) is then given by

$$f_g = \frac{1}{4\pi} \quad , \quad \Psi_1 = \Psi_2 \, e^{-1/2} \quad , \quad U_0 = \frac{\pi}{8} \frac{1}{Z_w f_0} \frac{\Psi_2^2 E_0^2}{e} \tag{4.7}$$

For a free-space dielectric medium this gives a characteristic impedance

$$Z_c = f_g Z_w \simeq 30\,\Omega \tag{4.8}$$

This is a somewhat higher impedance than has been typically used. Note that the impedance to be driven by the oscillator should be larger than this to support the quarter-wave resonance. Combining this requirement with the use of a transformer, one can choose the transfer function T (which can even be less than one, if desired) to give the desired combination of V_a and Q.

One cost of going to a higher Z_c is the requirement of higher V_0 (charge voltage) to attain this maximum stored energy. If one wishes a lower V_0, then other techniques may be more appropriate, such as varying the cross section design [5], at some expense of maximum stored energy.

5. Concluding Remarks

Now we can see some of the trades in achieving maximum performance from a switched oscillator. Note that the present analysis is limited by some simplifying assumptions. These include neglecting various losses in the switch, conductors, and antenna. The designer now has many options to consider.

References

1. C. E. Baum, "Combined Electric and Magnetic Dipoles for Mesoband Radiation, Part 2", Sensor and Simulation Note 531 May 2008.
2. C. E. Baum and J. M. Lehr, "Nonuniform-Transmission-Line Transformers for Fast High-Voltage Transients", Circuit and Electromagnetic System Design Note 44, February 2000; "Tapered Transmission-Line Transformers for Fast High-Voltage Transients", IEEE Trans. Plasma Science, 2002, pp. 1712-1721.
3. C. E. Baum, "Switched Oscillators", Circuit and Electromagnetic System Design Note 45, September 2000.
4. C. E. Baum, "A Transmission-Line Transformer for Matching the Switched Oscillator to a Higher-Impedance Resistive Load", Circuit and Electromagnetic System Design Note 46, August 2001.
5. C. E. Baum, "Some Thoughts Concerning Extending the Performance of Switched Oscillators", Circuit and Electromagnetic System Design Note 54, March 2008.