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The Peaking Circuit Revisited

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Abstract

The peaking circuit has been extensively used in NEMP simulators. The ideal output waveform for these simulators is a fast-rising, double exponential pulse. This paper develops a set of equations that can be used to calculate the value of the peaking capacitance and the switching time for a circuit with inductance and resistance in the three legs of the circuit. While this paper does not provide a closed-form solution for these two unknowns, it provides a set of simultaneous equations that can be solved using a calculator or a computer.

THE PEAKING CIRCUIT REVISITED

INTRODUCTION

The peaking circuit has been used extensively in EMP simulators. The ideal output pulse from these simulators is an overdamped, double exponential pulse with a rise time that is an order of magnitude, or more, shorter than the exponential decay. The pulse energy for these generators is usually stored in a Marx generator which has a substantial inductance. The peaking circuit is used to charge this inductance before the simulator is switched into its load.

The full first-order circuit of a pulse generator of this type is shown in Figure 1. The source element in the circuit, the Marx generator, is represented by elements C_1, L_1, R_1, and S_1. The discharge form this circuit is initiated by closing S_1. This redistributes the energy in the Marx in the reactive elements L_1, C_2, and L_2, and deposits some energy in the resistive losses in the circuit, R_1 and R_2.

The elements C_2, L_2, and R_2 comprise the peaking circuit. Switch S_3 closes when the peaking leg is charged to the required voltage. This energizes the load circuit represented by L_3 and R_3.

A circuit of this type was analyzed by Lupton[1] for the case where $L_2 = L_3 = R_1 = R_2 = 0$ and $C_1 \gg C_2$. Subsequently the circuit was analyzed by the author[2] under generally the same conditions, except that $C_1 \geq C_2$. Both these analyses gave the value of C_2 and the time delay between the closure of S_1 and S_2 for generating an exponential output pulse in the load. These solutions are in closed form.

The analysis described in this paper extends this earlier work. It develops a set of equations that can be used to determine the initial voltage on C_1, the values of C_1 and C_2, the time delay between switch closures and the output wave rise time for the full circuit shown in Figure 1. The solution is for the case where the circuit generates the required double exponential output pulse shape.

The resulting equations have not been solved in closed form. However, they can be solved numerically using a digital computer, a calculator or even a slide rule.

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1 Waveform Distortion from Peaking Circuit Switch Jitter, William H. Lupton NRL Memorandum Report 1829, November 1967 and Not 1 "Pulsed Electrical Power Circuit and Electromagnetic System Design Notes". AFWL TR 73-166, April 1973.

2 Solution of Peaking Equation for Finite Storage Capacitor Size, by John L. Harrison, Note 32 "Pulsed Electrical Power Circuit and Electromagnetic System Design Notes", January 1973.

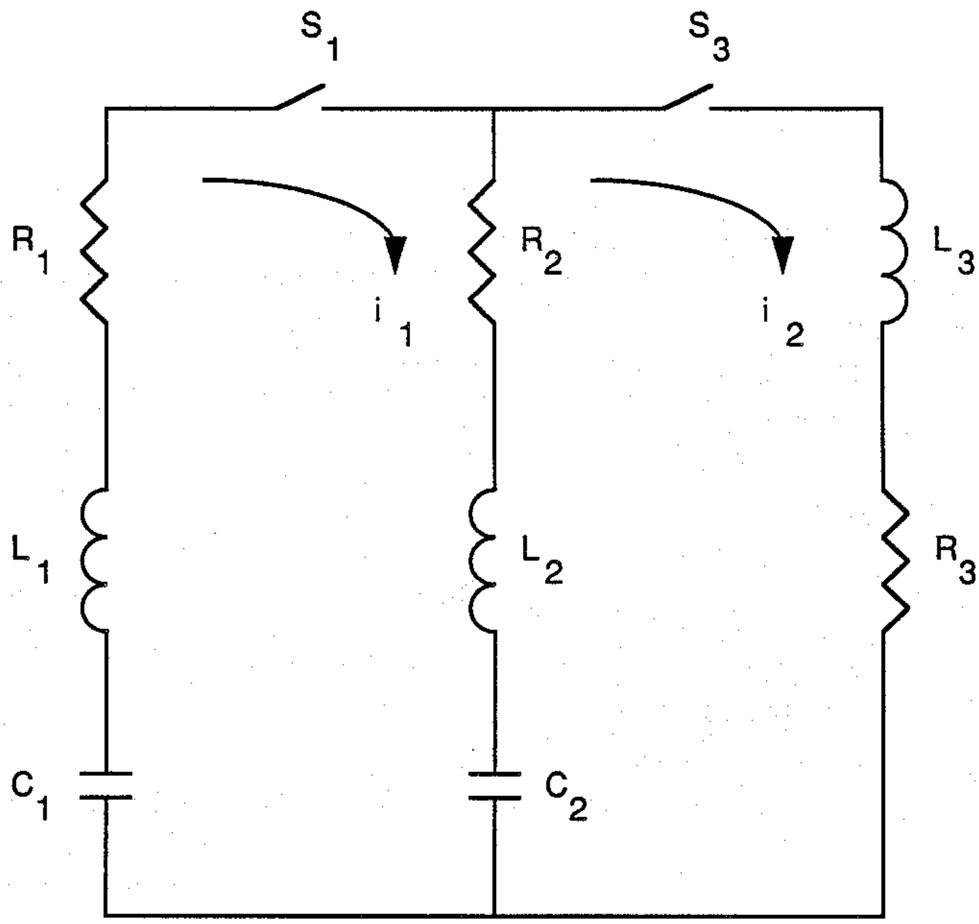

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Figure 1. Schematic of circuit analyzed

GENERAL OUTLINE OF THE METHOD USED

The equations for the Figure 1 circuit are analyzed separately for the time before switch S_3 closes, and the time after this switch closes. Real time, t, measured from the time S_1 closes, is used for the first analysis, and the retarded time

$$t' = t - t_3$$

is used for the second analysis. Here t_3 is the closure time of S_3.

The analysis is straightforward for the early time solution since this part of the circuit is a simple damped series LCR circuit. To solve the late time circuit, the differential equations for the circuit are developed, and the circuit is evaluated by forcing the required solution

$$i_2(t') = \frac{V_0}{R_3}\left(e^{-\frac{t'}{\tau_1}} - e^{-\frac{t'}{\tau_2}}\right) \tag{1}$$

and assuming

$$i_1(t') = A_1 e^{-\frac{t'}{\tau_1}} - A_2 e^{-\frac{t'}{\tau_2}} \tag{2}$$

where

V_0 = the specified output voltage

τ_1 = the specified decay time constant

τ_2 = is the time constant of the rise time which is determined by the circuit parameters.

Equations (1) and (2) are differentiated and integrated, and the resulting values are substituted in the differential equation of the current. The equations are expressed in terms containing exp $(-t'/\tau_1)$, terms containing exp $(-t'/\tau_2)$ and terms with no exponential. These three types of terms are orthogonal, so three independent sets of equations can now be written.

The equations developed using the above techniques, and the charge conservation equation

$$C_1 V_{C1}(0) = \frac{V_0}{R_3}(\tau_1 - \tau_2) \quad , \tag{3}$$

which equates the initial charge on capacitor C_1 to the total charge delivered to the load, are then used to determine:

- The initial voltage, $V_{C1}(0)$, on C_1
- The capacitance of C_1
- The capacitance of C_2
- The voltage on C_2 at time t_3
- The current, $i_1(t_3)$, around the initial loop at the time t_3
- The time constant, τ_2, of the output current rise time
- The time t_3, when S_3 closes.

It is assumed that the values of all other circuit elements are known.

DIMENSIONLESS QUANTITIES

The following dimensionless quantities will be used in the equations developed in the analysis

$$a_1 = \frac{\tau_1^2}{\tau_1^2 - R_1C_1\tau_1 + L_1C_1}$$

$$a_2 = 1 - \frac{L_2}{R_2\tau_1}$$

$$a_3 = 1 - \frac{L_3}{R_3\tau_1}$$

$$b_1 = \frac{\tau_2^2}{\tau_2^2 - R_1C_1\tau_2 + L_1C_1}$$

$$b_2 = 1 - \frac{L_2}{R_2\tau_2}$$

$$b_3 = 1 - \frac{L_3}{R_3\tau_2} \quad .$$

It will be seen that a_2 and a_3 are constants, and that the other quantities are variables since they include the variables C_1 and/or τ_2 in their formulation.

ANALYSIS OF CIRCUIT WITH S3 CLOSED

The circuit equations for times greater than t_3, the time when S_3 closes, are

$$V_{C1}(t_3) - L_1 \frac{di_1}{dt'} - R_1 i_1 - \frac{1}{C_1}\int_0^{t'} i_1\, dt'$$

$$= V_{C2}(t_3) - L_2 \frac{d(i_2 - i_1)}{dt'} - R_2(i_2 - i_1)$$

$$- \frac{1}{C_2}\int_0^{t'} (i_2 - i_1)\, dt'$$

$$= L_3 \frac{di_2}{dt'} + R_3 i_2 \quad . \tag{4}$$

As mentioned above, these equations are written in the retarded time

$$t' = t - t_3 \quad .$$

Equations (1) and (2) are now used to rewrite Equation (4) giving

$$V_{C1}(t_3) - \frac{\tau_1 A_1}{C_1}\left\{1 - \frac{e^{-\frac{t'}{\tau_1}}}{a_1}\right\} + \frac{\tau_2 A2}{C_1}\left\{1 - \frac{e^{-\frac{t'}{\tau_2}}}{b_1}\right\}$$

$$= V_{C2}(t_3) - \frac{\tau_1 - \tau_2}{R_3 C_2} V_0 + \frac{1}{C_2}(\tau_1 A_1 - \tau_2 A_2)$$

$$+ \left(\frac{V_0}{R_3} - A_1\right)\left(\frac{\tau_1^2 - R_2 C_2 \tau_1 + L_2 C_2}{C_2 \tau_1}\right) e^{-\frac{t'}{\tau_1}}$$

$$-\left(\frac{V_0}{R_3} - A_2\right)\left(\frac{\tau_2^{\,2} - R_2 C_2 \tau_2 + L_2 C_2}{C_2 \tau_2}\right) e^{-\frac{t'}{\tau_2}}$$

$$= V_0 \left\{ a_3 \, e^{-\frac{t'}{\tau_1}} - b_3 \, e^{-\frac{t'}{\tau_2}} \right\} . \tag{5}$$

Equating the exp ($- t'/\tau_1$) terms of Equations (5), we get

$$A_1 = \frac{a_1 a_3 C_1}{\tau_1} \, V_0 \tag{6}$$

and

$$C_2 = \frac{(\tau_1 - a_1 a_3 R_3 C_1)\, \tau_1}{a_3 R_3 \tau_1 + a_2 R_2 \, (\tau_1 - a_1 a_3 R_3 C_1)} . \tag{7}$$

and equating the exp ($- t'/\tau_2$) terms of Equation (5), we get

$$A_2 = \frac{b_1 b_3 C_1}{\tau_2} \, V_0 \tag{8}$$

and

$$C_2 = \frac{(\tau_2 - b_1 b_3 R_3 C_1)\, \tau_2}{b_3 R_3 \tau_2 + b_2 R_2 \, (\tau_2 - b_1 b_3 R_3 C_1)} . \tag{9}$$

Finally, equating the constant terms of Equation (5), and substituting the values of Equations (6) and (8) for A_1 and A_2, we get

$$V_{C1}\,(t_3) = (a_1 a_3 - b_1 b_3)\; V_0 \tag{10}$$

and

$$V_{C2}(t_3) = \left\{\frac{\tau_1 - \tau_2}{R_3 C_2} - (a_1 a_3 - b_1 b_3)\frac{C_1}{C_2}\right\} V_0 \quad .$$

The value of i_1 at time t_3 is obtained from Equations (2)

$$i_1(t_3) = \left(\frac{a_1 a_3 C_3}{\tau_1} - \frac{b_1 b_3 C_2}{\tau_2} V_0\right) \quad . \tag{11}$$

The two equations for C_2, Equations (7) and (9), reduce the number of independent variables in the above equations to two; since if the value of any one of the variables C_1, C_2, or τ_2 are known, all the equations can be solved. C_1 and τ_2 will be used as the independent variables.

ANALYSIS OF THE CIRCUIT BEFORE S3 CLOSES

Equation (3) can be rewritten to give the value of the initial voltage on C_1 in terms of the two independent variables C_1 and τ_2. The equations for the voltage V_{C1} on C_1 and current i_1 can then be formulated for the time t_3 when S_3 closes. Time t_3 then becomes a third independent variable.

The formula for $V_{C1}(t_3)$ and $i_1(t_3)$ gives us two equations in variables C_1, τ_2, and t_3 since the values must equal the values given in Equations (10) and (11). Thus, we now have three simultaneous equations with three independent variables. These equations are

$$\frac{(\tau_1 - a_1 a_3 R_3 C_1)\tau_1}{a_3 R_3 \tau_1 + a_2 R_2(\tau_1 - a_1 a_3 R_3 C_1)} = \frac{(\tau_2 - b_1 b_3 R_3 C_1)\tau_2}{b_3 R_3 \tau_2 + b_2 R_2(\tau_2 - b_1 b_3 R_3 C_1)}$$

from Equations (7) and (9);

$$(a_1 a_3 - b_1 b_3) = \frac{(\tau_1 + \tau_2)}{R_3 C_1}\left[1 - \frac{1}{\omega_0 Z_0 C_1}\left\{1 - e^{-\alpha\omega_0 t_3}\left(\cos \beta\omega_0 t_3 + \frac{\alpha}{\beta}\sin \beta\omega_0 t_3\right)\right\}\right] \quad ; \tag{12}$$

and

$$\frac{a_1 a_3 C_1}{\tau_1} - \frac{b_1 b_3 C_2}{\tau_2} = \frac{\tau_1 + \tau_2}{R_3 C_1 Z_0 \beta} \quad \cdot \quad e^{-\alpha\omega_0 t_3} \sin \beta\omega_0 t_3 \qquad (13)$$

from Equations (10) and (11) and the solution of the circuit equations before S_3 closes. Here

$$\omega_0 = \sqrt{\frac{C_1 + C_2}{(L_1 + L_2)\, C_1 C_2}}$$

$$Z_0 = \sqrt{\frac{(L_1 + L_2)\,(C_1 + C_2)}{C_1 C_2}}$$

$$\alpha = \frac{1}{2Q}$$

$$\beta = \sqrt{1 - \alpha^2}$$

$$Q = \frac{Z_0}{R_1 + R_2} \quad .$$

The above equations are more complex than they appear because the quantities a_1, b_1, b_2, and b_3 contain one or both of the variables C_1 and τ_2 in their formulation.

SOLUTION OF EQUATIONS

The above equations have been solved for a range of values of L_2, L_3, R_1, and R_2 for given values of V_{C1} (0), L_1, τ_1, and R_3. However, these solutions have not been reduced to a form that is suitable for inclusion in this paper. Thus, an analysis of the effects of losses in the initial loop and inductance in the peaking an load leg of the circuit on the value of the peaking capacitance and the initial charge on the energy store will not be reported in this paper.

CONCLUSIONS

The above analysis shows that the peaking circuit can be designed to generate a pure double exponential pulse in a real circuit with losses and inductance in all of the legs of the circuit.